Investment Patterns In UK Manufacturing Establishments[#]

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Abstract

In this paper we document the extent to which lumpy investment behaviour is present in UK plant-level data. For this purpose we use the Annual Business Respondents Database (ARD) from 1980 to 1992.

The aim of this paper is twofold. First, it describes an estimation method of capital stock at the establishment level, by asset, based on the ARD. The distinctive feature of this work is the treatment of leased assets. Second, it provides evidence on the extent of non convexities and irreversibility of investment by asset in the UK. It highlights that a large fraction of aggregate investment is accounted for by few establishments that are investing a lot. Furthermore, for each establishment, a large fraction of its investment activity over a long horizon is accounted for by a few large episodes. Significant differences emerge in the investment patterns by asset, where "buildings and land" are the most rigid asset, "plant and machinery" the most flexible, "vehicles" are a rigid but not irreversible investment.

Innovative contributions to the descriptive literature on this topic are: focus on the UK, disaggregate analysis by asset, statistics on net as well as gross investment rates.

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Capital Adjustment Patterns in Manufacturing Plants*

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A common result from altering several fundamental assumptions of the neoclassical investment model with convex adjustment costs is that investment may occur in lumpy episodes. This paper takes a step back and asks "How lumpy is investment?" We answer this question by documenting the distributions of investment and capital adjustment for a sample of over 13,700 manufacturing plants drawn from over 300 four-digit industries. We find that many plants do undergo large investment episodes; however, there is tremendous variation across plants in their capital accumulation patterns. This paper explores how the variation in capital accumulation patterns vary by observable plant and firm characteristics, and how large investment episodes at the plant level transmit into fluctuations in aggregate investment. *Journal of Economic Literature* Classification Numbers: D24, L6, E22.
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I. INTRODUCTION

Among Michael Gort's many contributions to economics is his early work using establishment-level data at the U.S. Census Bureau. Professor Gort realized early on that aggregate statistics mask important underlying

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dynamics that belie the aggregate changes, and that to truly understand the dynamics within industries, one has to examine the underlying micro data. In his 1963 paper [23], "Analysis of Stability and Change in Market Shares," Professor Gort explored the extent to which the market share of firms that make up published concentration ratios change over time. After all, if the concentration ratio for a particular industry remains high and stable over time, it does not necessarily imply that the industry is stagnant and controlled by a small handful of dominant firms. In fact, the industry could be extremely competitive with market share amongst the firms changing quite markedly, yet the published concentration ratios would not convey this information. To resolve this issue, which required access to firm-level data on market shares, Professor Gort utilized the raw micro data files at the U.S. Census Bureau. He was one of the first economists to exploit establishment-level data files at the U.S. Census Bureau for economic research. Moreover, 30 years later, Professor Gort returned to the U.S. Census Bureau to undertake a project that examined productivity growth and learning in new plants. In his 1993 paper [3], "Decomposing Learning by Doing in New Plants," coauthored with B. H. Bahk, Professor Gort examined how productivity evolves in new plants as they age. Again, this is a paper that underscores the importance of understanding the underlying microeconomic dynamics as they relate to aggregate economic changes.

Continuing in the tradition that Professor Gort helped establish, this paper also uses U.S. Census Bureau establishment-level data to gain a better understanding of an aggregate phenomenon, in this case, investment activity. This paper examines the capital adjustment patterns for a large sample of manufacturing establishments. This is an important area to examine since accurately modeling new capital investment at the micro and macro levels has proved elusive. In standard neoclassical investment models, assumptions, such as convex adjustment costs and reversibility, dictate that firms continuously and smoothly adjust their capital stock over time. While theoretically tractable, these models generally fail to adequately explain investment fluctuations [1, 8]. The disappointing empirical performance of these investment models has caused economists to reexamine the potentially unrealistic assumptions of convex adjustment costs and reversibility. Rothschild [28] argued early on that adjustment costs faced by plants and firms possess nonconvexities for a variety of reasons.[1] Another

[1] The sources of speculated nonconvexities in the cost of capital adjustment include increasing returns, the cost of the equipment, costs associated with disruption, and installation costs.

assumption in the standard models that is unrealistic is reversibility, an area that has received a great deal of attention in recent years.[2] Models which assume nonconvex adjustment costs and irreversibility possess solutions where firms occasionally adjust their capital in discrete bursts when the capital stock falls (rises) below (above) a trigger level, solutions which differ markedly from those of standard neoclassical models.[3]

While a growing number of studies suggest that capital adjustments may occur in lumpy episodes, the theoretical literature is well ahead of its empirical counterpart.[4] This is largely due to the scarcity of data sets that follow the investment process for a large number of establishments. This situation is changing as access to microeconomic data, in particular plant-level data, increases. For instance, Caballero, Engel, and Haltiwanger [10], Power [27], and Cooper, Haltiwanger, and Power [14] investigate the lumpiness of plant-level investment and its relationship to aggregate investment fluctuations using the plant-level data on investment from U.S. Census Bureau micro data files.[5] There are two main findings. First, investment by manufacturing plants is characterized by periods of intense investment activity interspersed with periods of much lower investment activity. Second, episodes of intense investment activity are responsible for a significant fraction of aggregate investment fluctuations.

This paper also examines the patterns of investment spending at the plant level and relies on the Census Bureau micro data. As compared to the Census-based research discussed above, this paper is more descriptive. The goal of this paper is to present a series of stylized facts that will serve as benchmarks for investment models. In particular, the goal of this paper is not simply to show whether investment is lumpy or not, but instead to focus on how the distributions of investment and capital adjustment vary by plant characteristics (e.g., industry, size, age, and ownership) and by level of micro-unit aggregation (plant, line-of-business, and firm). Finally, the paper relates the evidence on micro-level lumpiness to aggregate investment fluctuations.

[2] Reviews of investment models with irreversibility include Pindyck [26], Dixit [17], and Dixit and Pindyck [18].

[3] Other underlying assumptions in neoclassical models are that capital is homogeneous and capital depreciates geometrically. Feldstein and Rothschild [22] discuss the unrealistic nature of homogeneous capital and geometric decay, and how changing these assumptions can result in lumpy investment patterns.

[4] The literature which examines labor adjustments is more mature. The importance of large proportional adjustments in employment at the establishment level has been documented by Hamermesh [24], Davis and Haltiwanger [16], and Caballero, Engel, and Haltiwanger [10].

[5] In addition to the Census Bureau microeconomic data studies, there are a number of other studies that examine machine replacement at the micro level. Rust [29] examines replacement investment with bus engines, and Cooper and Haltiwanger [13] model retooling in automobile assembly plants.

We first examine the patterns of capital accumulation within plants and focus on the magnitude of capital adjustments at annual frequencies. We find:

(1) Many plants occasionally alter their capital stocks in lumpy fashions. Over half of the plants in our sample experience a 1-year capital adjustment of at least 37%. While many manufacturing plants experience episodes of intense investment activity, 80% of the plants in a given year change their net capital stock by less than 10%. These relatively small changes account for 52% of total sample investment.

Whether or not capital adjustment is "lumpy" depends on to what it is compared. To help quantify what "lumpy" means, we compare the results from the sample of plants to those generated by simulated investment models, where the simulations include possible S, s behavior by including trigger and target levels. The larger are the estimated trigger levels in the simulations, the "lumpier" is capital adjustment. We find:

(2) The simulation models that best fit the observed capital adjustment patterns are those that possess trigger levels substantially above and below zero. That is, the simulation results that best fit the observed data are those in which plants mainly invest when the difference between the desired and actual capital stocks is substantially different. Otherwise, plants usually invest in small amounts, amounts that could be related to replacement and maintenance investment.

Although many plants do experience a large investment episode, perhaps our most striking finding is the tremendous variance across plants in their capital adjustment patterns. We find:

(3) With respect to plant characteristics, smaller plants, plants that undergo a change in organizational structure (e.g., ownership change), and plants that switch industries have lumpier investment patterns.

Although investment is conducted at the establishment level, investment decisions are made at the firm level. Hence, while investment may be relatively volatile at the establishment level, investment may be smoothed at the firm level, which may be consistent with the large literature on the role of firm finance constraints. In fact, we find:

(4) Plant-level capital accumulation patterns are considerably lumpier than those computed at the line-of-business level, and the line-of-business level capital accumulation patterns are noticeably more discrete than those at the firm level.

Whether or not investment is lumpy also influences models of aggregate investment. Increasing attention has recently been placed on unraveling aggregate fluctuations by examining the distribution of micro changes (e.g., [7, 9, 10, 14, 16]). Bertola and Caballero [5] model firms making investment decisions in an uncertain environment and when investment is irreversible. In this model, firms do not continually invest, but invest in lumps; hence, aggregate fluctuations in investment are partially attributable to changing proportions of the population undergoing large investment episodes. To shed light on this issue, we examine how plant-level changes in capital and investment transmit to aggregate fluctuations in investment, focusing particularly on the role of investment spikes. We find:

(5) Large investment projects in a small number of plants greatly impact aggregate investment. For our sample, 25% of expenditures on new equipment and structures goes into plants that are increasing their real capital stock by more than 30%. However, these plants make up only 8% of the sample. For the population as a whole, investment is highly skewed. In 1977 and 1987 the 500 largest investment projects accounted for 35.7 and 32.1% of total manufacturing investment.

(6) Periods of large aggregate investment are due, in part, to changes in the frequency of plants undergoing large investment episodes, though not necessarily large percentage changes in capital adjustments.

The paper proceeds as follows. Section II describes the data and the patterns of capital adjustment observed in our data sets, and provides results of simulations used to benchmark the empirical patterns. This section also examines how capital adjustment patterns vary by producer characteristics and by level of aggregation. Section III discusses the correlation between large capital adjustments and fluctuations in aggregate investment. Section IV provides summary analysis.

II. PLANT-LEVEL CAPITAL ACCUMULATION PATTERNS

In this section we examine the patterns of plant-level investment and capital growth, focusing especially on those periods when plants undergo large changes in their capital stocks. The section presents some basic statistics on capital growth rates and investment, and examines how these patterns vary by plant characteristics such as industry, plant size, and unit of aggregation (e.g., plant, line of business, firm). Before proceeding with a description of the basic patterns, we briefly describe the data. A more thorough discussion of the data set can be found in Doms and Dunne [20].

The information on annual investment and capital growth is constructed from a panel data set of manufacturing plants for the period 1972–1988. The establishment-level data are drawn from the Longitudinal Research

Database (LRD), which is maintained at the U.S. Census Bureau and contains establishment-level production data from the Annual Survey of Manufacturers (ASM). The main data set contains a balanced panel of establishments from the LRD and covers the period 1972–1988. The balanced nature of the panel ensures that capital stocks for plants can be constructed using the perpetual inventory method. The resulting data set includes 13,702 manufacturing establishments. This sample is small relative to the manufacturing population, which ranges from 312,000 to 360,000 plants over the sample period. However, while the sample coverage in terms of number of establishments is relatively small, these establishment are on average quite large and account for a significant fraction of manufacturing investment, production, and employment. Table I presents some basic characteristics of this data sample. The establishments in the sample averaged over 500 employees and accounted for 55.4–61.1% of manufacturing investment, employed 39.3–44% of manufacturing workers, and produced 47.4–53.8% of manufacturing output over the 1972–1988 period. While not reported in this paper, we have also constructed a larger data set that allows for establishment births and deaths. In general, the results reported below hold qualitatively for plants that do not span the entire time period. These results are reported in Doms and Dunne [20].

In order to measure plant-level capital growth rates, a capital series must be developed for each plant. In this paper we use the perpetual

TABLE I
Sample Coverage by Year

Year	Investment coverage (%)	Labor coverage (%)	Production coverage (%)	Average employment
1973	58.6	43.5	53.0	598.7
1974	60.1	44.0	53.8	600.4
1975	58.8	44.0	52.7	551.8
1976	56.9	44.2	54.0	570.3
1977	57.1	43.5	53.1	588.2
1978	55.4	43.3	53.2	608.1
1979	59.3	43.2	53.7	622.2
1980	60.5	42.7	52.7	601.9
1981	60.5	43.2	52.6	595.8
1982	57.7	42.2	50.3	548.7
1983	61.1	41.9	51.1	534.7
1984	57.3	42.8	51.8	558.7
1985	60.8	42.9	50.6	547.6
1986	58.2	42.5	50.1	530.5
1987	56.7	40.2	48.3	520.7
1988	58.1	39.3	47.4	514.3

inventory method. The capital stock in period t for plant i, $K_{i,t}$, is defined as

$$K_{i,t} = K_{i,t-1}(1 - \delta) + I_{i,t}, \tag{1}$$

where δ represents the depreciation rate and $I_{i,t}$ is current period investment. The rate of depreciation, δ, is estimated for each three-digit industry by imbedding the depreciation parameter within a production function. The parameters of the production function are estimated simultaneously with the parameters of the investment stream (see Doms [19] for details). Utilizing the above measure for the capital stock we construct net capital growth rates analogous to the employment growth rates of Davis and Haltiwanger [16]. The growth rate of capital for plant i at time t is computed as

$$GK_{i,t} = \frac{I_{i,t} - \delta K_{i,t-1}}{0.5 \cdot (K_{i,t-1} + K_{i,t})}. \tag{2}$$

For each plant in our sample, we compute $GK_{i,t}$ for every year from 1973 to 1988.[6,7]

Figure 1 presents two distributions, the density of $GK_{i,t}$ and the density of $GK_{i,t}$ weighted by $I_{i,t}$. The figure shows that 51.9% of plants in a year increase their capital stock by less than 2.5%, while 11% of plants in a year increase their capital stock by more than 20%. However, the few plants that do undergo large changes contribute significantly to the level of aggregate investment. The weighted distribution shows that 25% of investment is in plants increasing their capital stock by more than 25%. At the other end of the distribution, 19.2% of investment is occurring in plants changing their capital stock by less than 2.5%.

Figure 1 shows that the distribution of investment is skewed, with a small number of plants accounting for a relatively large share of investment. While this is present in our subsample of data, it is also true in the establishment population as a whole. Table II gives the share of total investment in 1977 and 1987 accounted for by the top 100 investing plants, top 500 investing plants, top 1000 investing plants, etc. Also given in Table II are the analogous figures for ranked employment and output. This table

[6] The Annual Survey of Manufacturers stopped collecting the book value of capital data in 1989, as well as other investment related data, making it difficult to compute capital stocks after 1988.

[7] Unfortunately, the above expression ignores early retirements in the construction of the capital stock. The LRD does contain some data on retirements, but these data appear to contain significant errors. The constructed growth rate is therefore a relative measure of new capital accumulation net of depreciation.

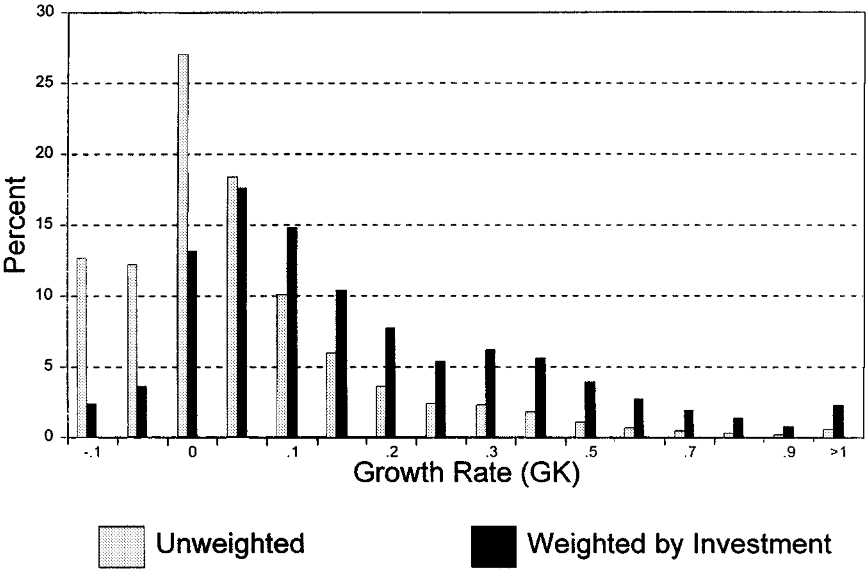

FIG. 1. Capital growth rate (*GK*) distributions: Unweighted and weighted by investment.

is based on the *entire* manufacturing establishment population. The overall message is relatively clear. A small number of plants account for a large fraction of investment. In 1977 and 1987, 18.2 and 16.2% of total manufacturing investment was accounted for by the top 100 plants, respectively. In contrast, the top 100 plants accounted for a substantially smaller fraction of ouput (9.0%) and employment (5.9%). Note that 100 plants make up only 0.028% of the entire population. The bottom line is that in a cross section a small number of investment "projects" account for a substantial fraction of aggregate investment. While this cross-sectional result is suggestive of "lumpy" investment, it does not provide any information on the within-plant investment patterns over time. It is a description of these within-plant patterns of investment and capital adjustment that we turn to next.

To examine the within-plant capital accumulation patterns, we construct two sets of ranks to describe the distributions of capital growth and investment at the plant level. The first measure constructs a ranked distribution of capital growth rates for a plant. For each plant in the balanced panel, we rank their capital growth rates from highest to lowest, so that their maximum growth rate is rank 1 and their lowest growth rate is rank 16. Throughout this paper, the rank 1 growth rate is denoted by MAXGK. Figure 2a presents the means and medians of these ranked

TABLE II
Share of Investment, Employment, Shipments, and Capital Accounted for by the Top Plants in Each Category

	Investment	Employment	Output	Capital stock
	1987 Census of manufactures: 358,567 plants			
Top 100 plants	.16204	.06344	.10077	.11888
Top 500 plants	.32154	.14057	.23031	.28882
Top 1000 plants	.41268	.18982	.30819	.38497
Top 5000 plants	.64769	.36233	.52581	.60963
Top 10000 plants	.74987	.47020	.62994	.70622
Top 25000 plants	.86863	.64043	.77045	.83002
Top 50000 plants	.93531	.77445	.86831	.90761
	1977 Census of manufactures: 350,648 plants			
	Investment	Employment	Output	Capital stock
Top 100 plants	.18172	.05932	.09005	.12883
Top 500 plants	.35657	.14584	.21638	.29359
Top 1000 plants	.44948	.20269	.29398	.39090
Top 5000 plants	.67240	.38958	.51551	.62407
Top 10000 plants	.76821	.50301	.62389	.72395
Top 25000 plants	.87931	.67753	.77172	.84548
Top 50000 plants	.94131	.80945	.87187	.91819

growth rates, so the first set of bars in Fig. 2a shows the mean and median MAXGK. The next set of bars shows the means and medians of the second largest growth rates, and so on. These bars indicate that the mean MAXGK slightly exceeds 46%, while the median is 36%. The means and medians drop off significantly after rank 1. Figure 2a illustrates that many plants experience a few periods of intense capital growth and many periods of relatively small capital adjustment: of the 16 capital growth rate ranks, 12 possess means or medians between -10 and $+10\%$.

Besides the growth rates of the capital stock, we are also concerned with episodes of investment that account for a large share of a plant's total investments. Figure 2b plots the mean proportion of total 16-year investment that occurs in each year. For instance, the leftmost bar represents that the average plant experiences a 1-year investment episode that accounts for 24.5% of its total real investment spending over the 16-year interval. The secondary growth rate accounts for 14.7%, and the third highest accounts for 10.9% of investment. This implies that, on average, half of a plant's total investment over the 1973–1988 period was performed in just three years. An important point is that while a significant portion of investment occurs in a relatively small number of episodes, plants still invest in every period.

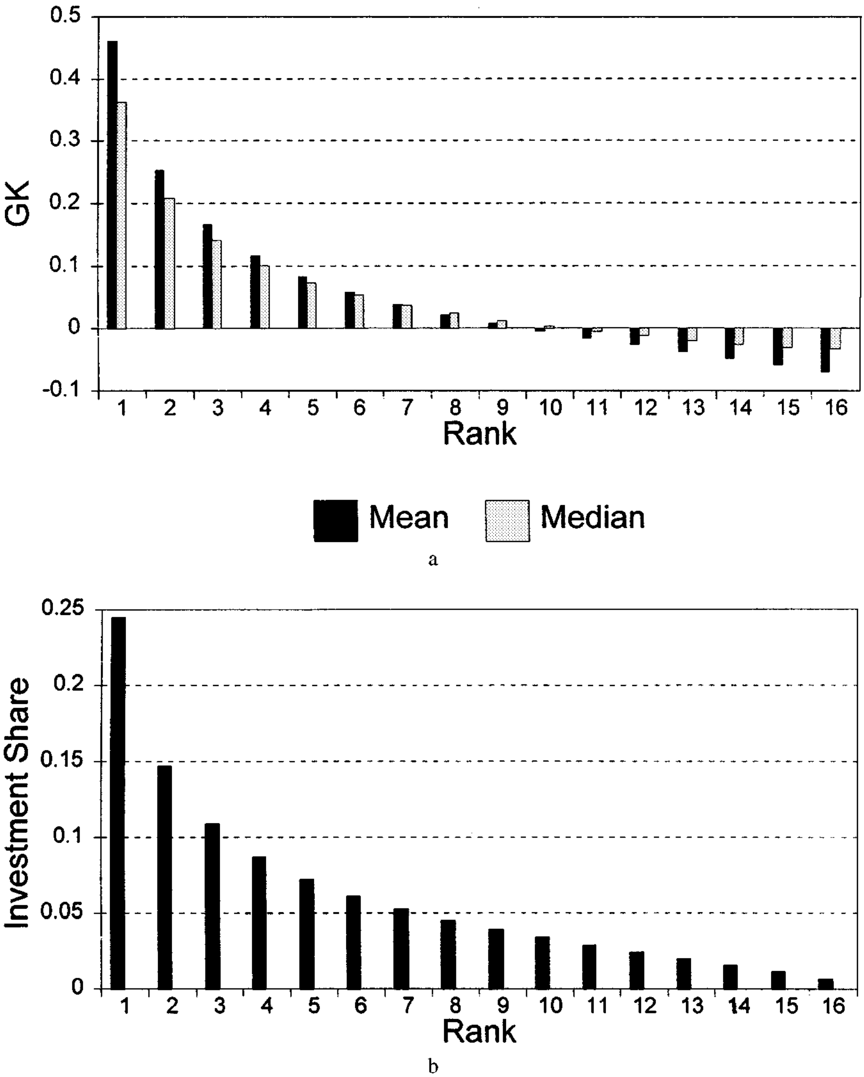

FIG. 2. Capital growth rates (*GK*) by rank, means, and medians. (b) Mean investment shares by capital growth rate rank.

What does Fig. 2a and b say about whether investment is "lumpy" or not? By construction, these figures slope down, and it is difficult to tell if, for instance, the data generating the figures come from something as simple as a Gaussian white noise process or whether the data are truly representative of a "lumpy" process. To benchmark our results, we compare our empirical results to simulations of simple capital investment models that include the possibility of lumpy adjustment episodes. Although the simulations do not formally test particular investment models, the simulations do provide a convenient benchmark to view our results. The following model was kindly provided by Jeffrey Campbell.

Let $k^*_{i,t}$ denote the optimum level of the logarithm of the capital stock of plant i at time t if the plant faced no frictions in adjustments, frictions that might arise from nonconvex adjustment costs or irreversibilities. Let $k_{i,t}$ be the actual capital stock. In the simulations that follow, we assume that the optimum level of capital, $k^*_{i,t}$, follows a random walk of the form

$$k^*_{i,t} = k^*_{i,t-1} + \varepsilon_{i,t}, \qquad \varepsilon_{i,t} \sim N(\mu, \sigma^2). \tag{3}$$

The disturbance $\varepsilon_{i,t}$ is i.i.d. across time and plants. Let $z_{i,t} = k_{i,t} - k^*_{i,t}$ be the difference between the frictionless optimum and the actual capital stock. The investment decision for a plant follows that of a general S, s model, where the trigger levels are denoted by U and L and target levels by u and l, such that $L \le l \le u \le U$. For instance, if $U = u = l = L = 0$, then there are no frictions and plants will always invest to their optimal frictionless level of capital and capital adjustment would be normally distributed. However, if the target levels do differ from zero, then there will be periods when no investment takes place, that is, periods in which $z_{i,t-1} + \varepsilon_{i,t}$ lies within the U, L band. If $z_{i,t-1} + \varepsilon_{i,t} < L$, then the plant will invest up to l. Likewise, if the optimum level of capital falls sufficiently, $z_{i,t-1} + \varepsilon_{i,t} > U$, then the plant will disinvest to u. We modify this basic friction model by adding replacement investment since some investment always takes place in our sample of establishments. Replacement investment, $r_{i,t}$, is uniformly distributed and is independent across time and plants.

The simulations are performed with 1000 plants and are run for 300 periods. The last 16 observations for each plant are taken and the capital adjustments are ranked, just as they are ranked with the real data. The parameters of the simulations are calibrated to minimize the mean squared error between the simulated values and the real values of the ranked capital adjustments. For nearly all of the simulations presented in this paper, the values of the replacement and innovation parameters are nearly identical; for the innovation parameters, $\mu = 0.05$ and $\sigma = 0.18$. The mean value of replacement investment is 0.05 with a standard deviation of 0.005–0.02.

Figure 3 reproduces Fig. 2a with the results of two simulations. The first simulation is the best fitting simulation with frictionless adjustment, $U = u = l = L = 0$. What is perhaps most striking about the frictionless adjustment simulation is its symmetry, which stands in stark contrast to the asymmetry in the real data. Additionally, the frictionless simulation does not drop as quickly or have as many periods with low capital accumulation activity as in the real data. The second simulation presented in Fig. 3 introduces frictions, that is, the target and trigger levels are allowed to deviate from zero. The friction parameters that produce the best results are $L = -0.34$, $l = -0.05$, $u = 0.20$, and $U = 0.22$. What is most striking about this simulation is how the simulated values sharply fall after the first rank and then stay much closer to 0, as in the real data. In fact, the mean squared error between the simulations and the actual data falls from 0.129 for the frictionless model to 0.012 for the friction model.

The results in Fig. 3 are for the entire sample of establishments that span the sample period. What is also striking is how the results in Fig. 2a vary by other observable plant characteristics, such as size. Figure 4 presents mean ranks by size quartile, where plants are ranked by their mean employment over the sample period. The basic result is that smaller plants have higher maximum growth rates than the largest plants. We again perform the simulations for these four plant size categories, and the parameter that changes the most is the trigger level L, which goes from -0.37 for the smallest quartile to -0.20 for the largest quartile, a significant difference. One of many possible reasons why smaller establish-

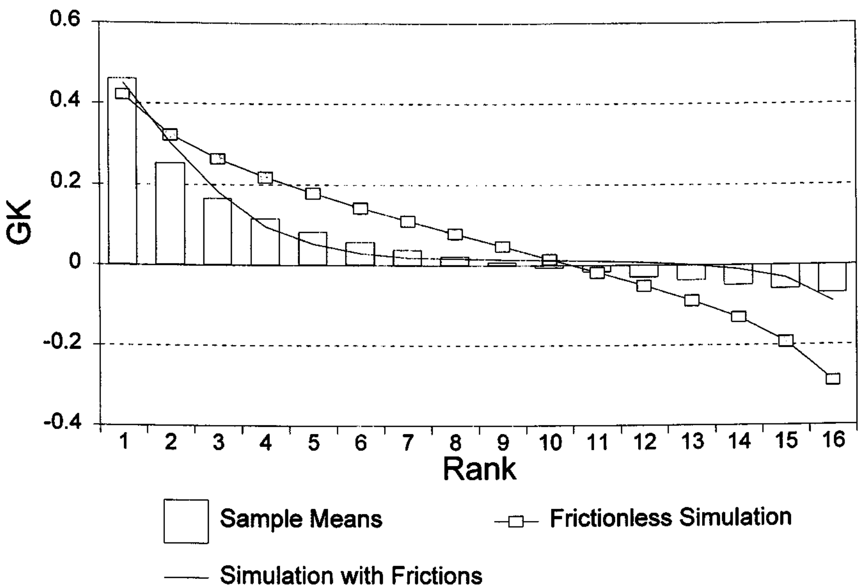

FIG. 3. Mean capital growth rates (GK) by rank, sample means, and simulated values.

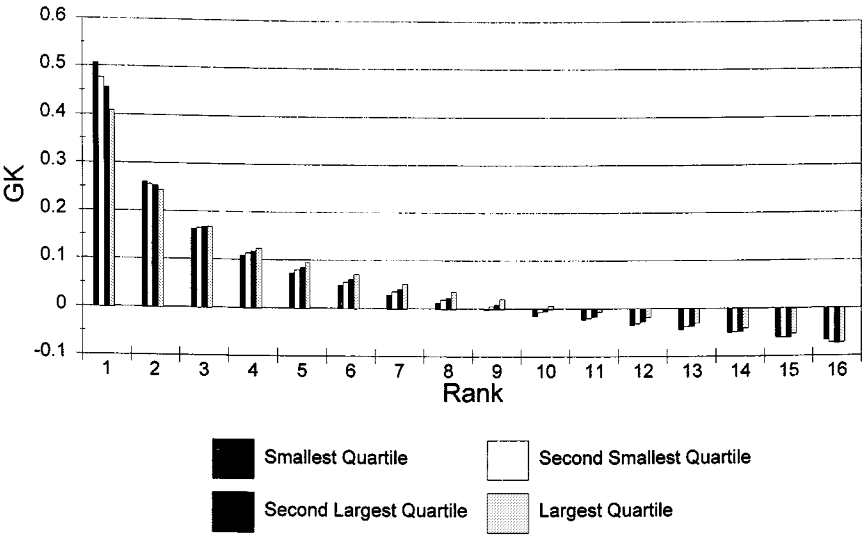

FIG. 4. Mean capital growth rates (GK) by rank and by size quartile.

ments may have higher trigger levels than larger establishments may be the indivisible nature of capital equipment; buying a single new machine at a smaller plant may represent a large share of its capital stock, so that its investment pattern may appear "lumpy." Large plants employing many machines may have smoother investment patterns because a single machine is a very small share of its capital stock. Additionally, one could view a large plant as a collection of smaller operations producing a range of products. These multiproduct operations may face less variable sales due to the fact they produce a number of different products, and hence their investment may be smoother as well.

Up to this point the unit of observation has been the plant; however, there are many arguments which suggest that the investment decisions of a plant are made at the divisional or firm level. Additionally, there are reasons why firms may smooth investment across plants. To examine how capital adjustment patterns vary by plant and firm, we construct capital adjustment ranks at the plant, the two-digit industry line-of-business level, and the firm level. The sample used to construct the plant, line-of-business, and firm statistics is a subset of the balanced panel. First, only those plants that remain with a single firm for at least 14 out of the 16 years are used. Second, only those plants that belong to firms with at least three plants are kept. Given these requirements, only 5822 plants out of the 13,702 plants in the balanced panel remain, representing 648 firms and 955

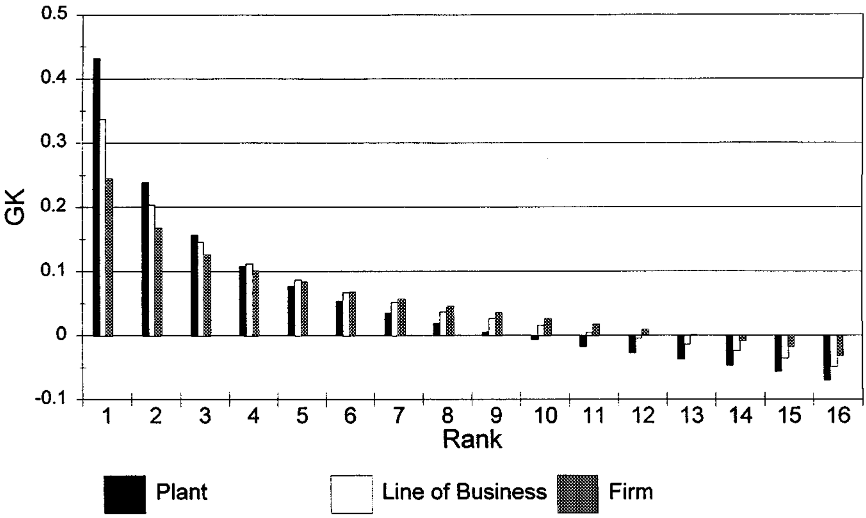

FIG. 5. Mean capital growth rates (*GK*) by plant, line of business, and firm.

lines of business. Note, however, that these plants make up 72.5% of the balanced panel investment.

The results of this exercise are presented in Fig. 5.[8] Basically, the higher is the level of aggregation, the smoother is the capital adjustment rank distribution. Examining the height of the largest capital adjustment episode, the mean MAXGK for plants is 0.432, and it falls to 0.336 for the line of business, and falls even further to 0.245 for firms. Again, simulations for these three distributions are run, and the estimate for the trigger level L changes from -0.35 for plants, to -0.19 for the line of business, and to -0.10 for the firm. This finding of smoother investment at the firm level may be consistent with the results reported by Cummins, Hassett, and Hubbard [15]. Note, however, that the asymmetry of the capital growth rate distribution still persists even at the firm level.

The analysis, so far, shows considerable across plant variation in capital growth rates, suggesting some plants experience relatively smooth changes in their capital stocks while other plants undergo sizable jumps in their capital stocks. A possible explanation of the differences in size is that for

[8] The basic results in Fig. 5 also hold for the investments distribution. Examining the height of the largest investment spike episode, the mean plant maximum investment share is 24%. This is quite close to that reported in Fig. 2b for the entire balanced sample. The mean maximum plant investment share drops to 17.1% at the line-of-business level and to 15.8% at the firm level. The bottom line is that firm-level investment patterns appear to be considerably smoother than plant-level investment patterns.

some industries investment is inherently lumpy because of the nature of the capital goods (which could arise due to the indivisibility of large machines), while for other industries it may be easier to adjust capital more smoothly.[9] To examine this possibility, we model MAXGK for a plant as a function of size, controlling for industry and other effects.

We estimate a regression model using all plants in our balanced panel. Our plant-level measure of capital lumpiness is the maximum single year capital growth rate (MAXGK), which our simulations show to be closely related to the magnitude of the trigger levels. Also, we have constructed other variables that characterize a plant's capital adjustment patterns, and arrive at the same qualitative results. The regressions include controls for both plant and firm size. Plant size is modeled using a set of dummy variables representing plant-size quintiles. The quintiles go from smallest to largest, with the quintile representing the largest plants omitted. The firm size variables are similarly defined. Two variables are included to capture potential changes in organizational structure and production mix that may affect capital accumulation patterns. The first variable is a dummy variable indicating whether a plant has changed ownership during the sample period. The second variable is a dummy variable which indicates whether the plant changes the two-digit industry in which the plant operates. Two age variables are included to capture differences in the age of plants that entered the panel in 1972. Finally, the regressions are all run with four-digit industry dummy variables. To conserve on space, the industry coefficients are not reported in the tables.

The second column of Table III reports the regression results. The starkest result is the strong inverse relationship between plant size and MAXGK. Smaller plants have considerably larger spikes, even after controlling for industry and other plant characteristics. Alternatively, there is no discernible pattern in the firm size coefficients. The two variables which capture change in ownership and change in industry indicate that plants which undergo ownership changes or switch industries experience somewhat larger MAXGKs. This is consistent with the view that organizational and industry changes lead to changes in plant-level operations which affect capital accumulation decisions. In terms of the simulation models, changes in ownership structure and industry may be indicators of discrete changes

[9] Doms and Dunne [20] report considerably more industry-level detail. For example, in the case of investment spikes, we find that 10% of industries (four-digit SIC) have maximum investment spikes under 0.20, 80% have maximum investment spikes between 0.20 and 0.30, and the remaining 10% have maximum investment spikes exceeding 0.30. Hence, the investment spike patterns observed in Fig. 2b are also present in a wide range of four-digit SIC industries. The same finding would be true for the capital growth rate distributions. Figure 2a is qualitatively similar to the growth rate distributions for a large number of industries.

TABLE III
Capital Growth Rate Regression: MAXGK Is the Dependent Variable

440 4-digit Industry Controls	Included
16 Year Dummies in which MAXGK occurs	Included
Plant Size Quintiles (Smallest to Largest)	
1st Quintile	.319 (.012)
2nd Quintile	.178 (.011)
3rd Quintile	.109 (.010)
4th Quintile	.056 (.010)
5th Quintile	omitted
Firm Size Quintiles (Smallest to Largest)	
1st Quintile	.007 (.010)
2nd Quintile	.037 (.009)
3rd Quintile	.013 (.009)
4th Quintile	.014 (.014)
5th Quintile	omitted
Industry Change Indicator	.031 (.011)
Ownership Structure Change Indicator	.040 (.006)
1963 Age Dummy	−.086 (.009)
1967 Age Dummy	−.049 (.011)
Mean of Dependent Variable	.461
Number of Observations	13072
R^2	.211

in the desired capital level. On the other hand, older plants have generally smaller than average capital growth rate spikes. This last result is consistent with the Jovanovic's [25] model of industry evolution that predicts that the variance of growth should decline as firms age.

The regression coefficients provide basic evidence of how capital growth varies with observable plant characteristics. However, on the whole, the plant and industry characteristics explain relatively little of variation in the standard deviation of capital growth or in the size of MAXGK. The amount of variation explained by plant and industry controls is about 20%. In general this lines up with the results reported by Davis and Haltiwanger [16], who report R^2s of similar magnitudes for employment growth regressions.

III. AGGREGATE INVESTMENT FLUCTUATIONS

This paper has so far focused on the predominance of large capital adjustments in plants and the variation across plants in their capital adjustment patterns. Increasing attention has recently been placed on unraveling aggregate fluctuations by examining the distribution of micro changes (e.g., [7, 9, 10, 12, 14, 16]). In this section, we present some basic summary statistics on the relationship between aggregate fluctuations in investment, the uniformity of changes in capital, and the frequency of large capital adjustments.

Using the balanced panel, which annually accounts for approximately 58% of aggregate investment, we compute the frequency of plants that have their MAXGK and MAXI (the maximum investment share) in a given year. Figure 6 presents these frequencies in addition to aggregate real investment over the period 1973–1988. There are several items to note. The first is that the correlation between MAXI and aggregate investment is 0.59, which is significant at the 99% level. The correlation between MAXGK and aggregate investment, however, is not statistically significant. This is due primarily to the high frequency of MAXGKs in 1973 and 1974 which is not reflected in the aggregate data.

Figure 6 conveys that aggregate fluctuations are correlated with the frequency of plants undergoing large investment episodes. An alternative way to summarize the relationship between aggregate investment and lumpy episodes is to see if investment is more skewed or concentrated in high investment periods. To address this issue, we compute a Herfindahl index for investment in each year and plot this series in Fig. 7 along with

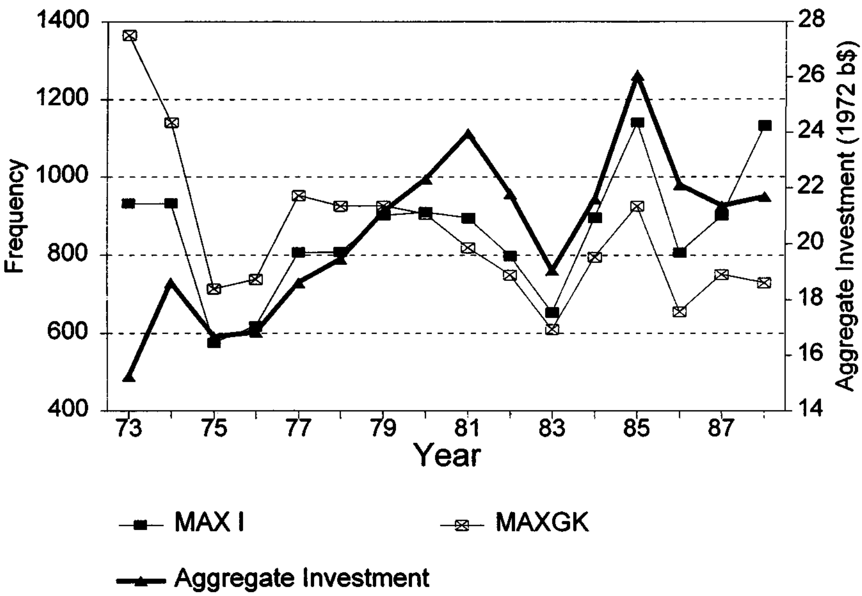

FIG. 6. Aggregate investment and frequency of plant spikes.

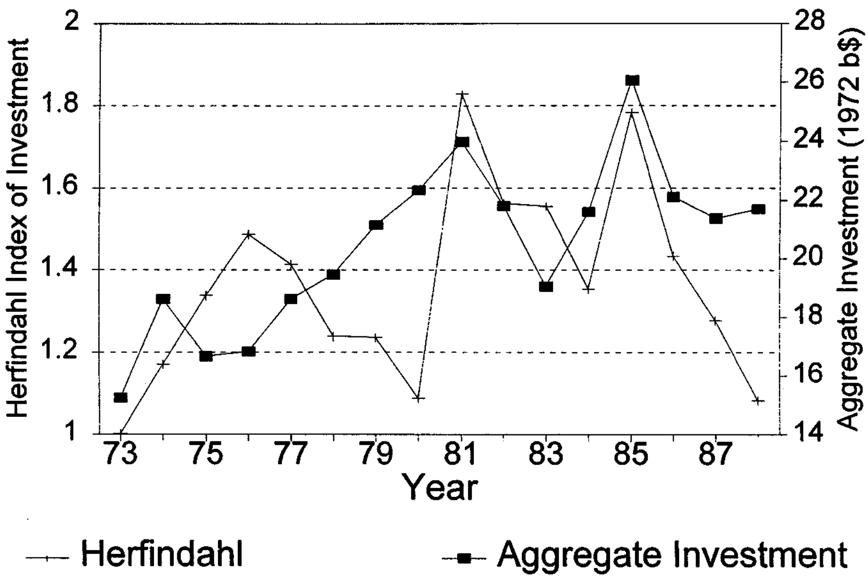

FIG. 7. Aggregate investment and the Herfindahl index of investment.

the aggregate investment series for the period 1973–1988.[10] In general, the series move together. The correlation between the two series is 0.450 and is significant at the 90% level. An interesting feature to note in Fig. 7 is that in 1980 and 1988 there are periods of relatively high aggregate investment in which there are relatively low Herfindahls. However, the two highest Herfindahls are in the 2 years with the highest aggregate investment.

IV. CONCLUSION

The objective of this paper is to present a series of stylized facts concerning the capital accumulation patterns for a large set of manufacturing plants. Although this paper is primarily descriptive in its examination of plant-level investment behavior, the facts presented here are quite striking and raise a number of issues. We have shown that many manufacturing plants do indeed alter their capital stocks in lumpy fashions, and these large adjustments do account for a significant portion of a plant's

[10] The Herfindahl index for investment is constructed as $\sum (I_i/TI)^2$, where I_i is investment in plant i and TI is aggregate investment. The Herfindahl is just the sum across all plants of the squared investment shares.

total capital expenditures and aggregate investment. However, we also find tremendous heterogeneity in the capital accumulation patterns across plants, finding that the degree of lumpiness of capital adjustment varies considerably across plants. These facts certainly raise the question of whether traditional representative agent models based on convex costs of adjustment are adequate enough to examine the dynamics of investment and capital accumulation.

That said, there are many features of the capital accumulation process that have not been addressed in the paper. One key aspect we have not examined is the within-plant timing pattern of investment. In particular, we have said little about what happens to a plant before a spike and, more importantly, what happens to a plant after a spike. To shed some light on this issue, Fig. 8 presents the mean growth rates of capital over a 5-year period surrounding the maximum capital growth spike, MAXGK. One can see that both before and after a spike, plants return to a much lower level of investment spending. This confirms the view that large capital growth episodes are interspersed with periods of relatively modest capital growth at the plant level. The specifics of investment timing are addressed more fully in papers by Caballero, Engel, and Haltiwanger [10], Cooper, Haltiwanger, and Power [14], and Power [27]. Importantly, Cooper, Haltiwanger, and Power [14] show that the probability of an establishment undergoing an investment spike increases in the time since the last investment spike. This line of research lends support to the notion that plants wait until their

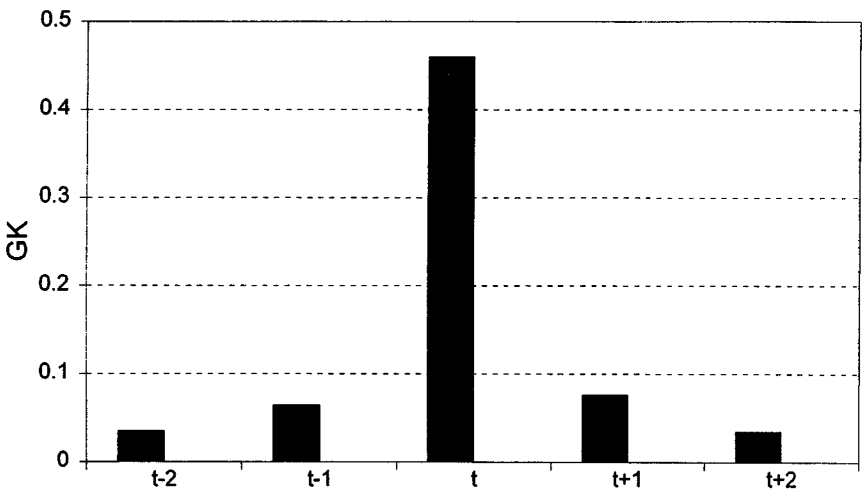

FIG. 8. Mean pre- and post-spike capital growth rates (*GK*).

actual capital stock deviates from the desired stock by a threshold before they invest.

In closing, this paper has described the patterns of capital accumulation using micro data on manufacturing establishments from the U.S. Census Bureau. This type of work builds on the tradition which Michael Gort helped establish almost three and half decades ago in his research using firm-level Census data from the 1940s and 1950s, and it highlights the importance of access to and development of micro data resources in understanding the underlying micro dynamics of aggregate data fluctuations.

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1. Introduction

Investment is one of the most volatile components of GDP. As a consequence, understanding investment activity has often been a considerable challenge for economists. Standard models of investment, where a representative firm is supposed to react to the user cost of capital do not fit aggregate data very well. The same applies to more sophisticated models that allow for convex cost of adjustment. As a consequence of the empirical failure of these standard models, a considerable amount of attention has recently been devoted to models that consider non-convex cost of adjustment. The attractive feature of these models is that they predict lumpy investment activity and periods of zero investment. These features seem to be important in micro data. Moreover, once one takes into account lumpy and inertial behaviour, aggregation problems become extremely important. Models with lumpy and inertial behaviour are able to generate very complex dynamics in the aggregate and, in particular, a slow adjustment to changes in 'fundamentals' and large volatility of aggregate investment.

It is therefore important to check that lumpy and inertial behaviour does characterize individual investment behaviour. Doms and Dunne (1998) have reported a number of statistics derived from plant level data for the US manufacturing sector that indicate that investment episodes are, indeed, concentrated and large. These authors, however, also report that the frequency of investment exactly equal to zero is quite small. Many firms report small amounts of investments, which are probably related to maintenance or replacement.

In this paper, we document the extent to which lumpy investment behaviour is present in UK plant-level data. For this purpose we use a newly available data set: the Annual

Business Respondents Database (ARD). Using the ARD, we characterize several aspects of the cross sectional distribution of investment and investment rates.

The ARD contains detailed information on investment flows by asset (plant and machinery, buildings and land, vehicles); it specifies for each of them the amount of acquisitions, disposals, assets not yet in production, leased assets[1]. However, unfortunately, ARD does not contain information on the stock of capital. For this reason, in the first part of the paper we describe how we estimated, using a perpetual inventory method, the capital stock at the plant level. A distinctive feature of our procedure is the treatment of leased assets. We discuss several checks on the validity of our procedure, some of which we present in the Appendix..

In the second part of the paper we move on to study the cross sectional distribution of investment episodes. In particular, as Doms and Dunne (1998), we document the degree of skewness of investment episodes. We show that investment does come in spurts that lead to large and concentrated episodes. While inspired by Doms and Dunne (1998), our analysis is however distinct from theirs for several reasons. First, and most obviously, we analyze UK rather than US data. Second, and more importantly, our analysis considers different components of investment separately. We show that this level of disaggregation matters for the prevalence of zeros, spikes of investment episodes and so on.. Third, our data set is unique in that it contains information on disposal (and not only on net investment). This information allows us to consider the importance of irreversibility in investment. Finally, we do not consider a balanced sample and, unlike Doms and Dunne (1998) we have small firms as well as large ones and we have entries and exits.

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[1] From 1988 to 1991 only.

The paper is organised as follows: section 2 describes briefly the data and gives some information on the procedure we used to estimate the value of the capital stock at the plant level. Details on this procedure, as well as evidence on the reliability of our estimates are relegated to the appendix. Section 3 is the core of the paper and reports our evidence on the lumpiness of investment behaviour. This section shows the likely importance of non-covex costs of adjustment, especially for some assets such as building and structure. Section 4 concludes.

2. Data and Capital Stock Estimates

2.1. The ARD Dataset

In this paper we use the Annual Business Inquiry Respondents Database (ARD). ARD is designed as a series of yearly cross sections of establishments, randomly drawn from the population of UK manufacturing establishments. The series starts in 1970; the last available year is 1996. An establishment is defined as the smallest unit that can provide all the data required by the questionnaire and is identified by the respondent itself.

The stratified sample includes all establishments above 100 employees, 50% of establishments employing 50-100 workers, 25% of establishments employing 20-50 workers, and no smaller establishments[2]. Only production establishments are included in the ARD, and they report only on their production activities. ARD covers private as well as government activities and non-profit bodies.

[2] Although the sampling probabilities have been modified in some years.

The questionnaire includes several hundred of questions, and it varies over time, focusing on different topics in different years. There is, however, a core of questions that is repeated each year. This includes questions about employment, investment, sales. There are no fiscal or financial details[3].

Unfortunately, there is no measure of the capital stock at the establishment level. The survey does include, however, detailed measures of capital flows: acquisitions, disposals, leased assets, assets not yet in production. The information on investment is divided by type of asset (plant and machinery, land and buildings, vehicles). For most of what we want to do an estimate of the capital stock at the plant level is necessary. Therefore we need to impute the capital stock in each year from the information available in the data set and from some additional pieces of information derived from alternative data sources. We describe our approach in the appendix. In the next section we provide a brief summary of the methodology applied and of the results we obtain.

A foreword about the sample selection we chose in this work is necessary. We analyse only a sub-sample of the entire data set. In particular, we select the period 1980-1992[4], leaving the analysis of the extended data set to future research. Further, the focus of this paper is the behaviour of *private* companies that declare to be active in the year[5]. Therefore, we exclude all the plants that do not satisfy these criteria. We end up with more than 181,000 observations on 13 years; about 13,000 establishments are included in

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[3] A more detailed description of the dataset, and of the variables included in it, can be found in Griffith (1999), Reduto dos Reis (1999).

[4] We start in 1980 because the establishment id-code changed in that year; this breaks the longitudinal archive in two sub-periods. We stop in 1992 because the change in the industry classification from SIC80 to SIC92 breaks the available aggregate investment series (see below for details).

[5] We also re-code the year according to the declared reporting period, that might be different from the calendar year in which the questionnaire was collected.

our sample every year. Table 11 in the appendix reports detailed information on the size and composition of our sample.

2.2. Estimating the capital stock

As mentioned above, our first task is to estimate the capital stock for each establishment in our sample. For such a purpose we use the perpetual inventory method: given an estimate of the capital stock in plant *i*, in asset *j* at time *t-1*, K_{it-1}^{j}, we obtain an estimate of K_{it}^{j} using the information on net investment in the specific asset I_{it}^{j} according to the following equation

$$K_{it}^{j} = I_{it}^{j} + \left(1 - \delta^{j}\right) K_{it-1}^{j} \quad (1)$$

where δ^{j} is the asset specific depreciation rate.

Having a series of observations on each establishment, we need to guess the level of the initial capital stock K_{i0}^{j} when the establishment enters the dataset in t=1. Given an initial value, and an assumption about the depreciation rate, δ^{j}, we can then apply equation (1) recursively. The panel dimension of the archive is obviously relevant for this purpose. We discuss the point extensively in the appendix.

To obtain an estimate of the initial capital stock of each establishment in the sample (K_{i0}^{j}), we impose that the capital stock in t=0 of an establishment that is first observed in t=1 is a share of aggregate (i.e. in the *population*) capital stock in t=0:

$$K_{i0}^{j} = \frac{L_{i0}}{L_{S0}} K_{S0}^{j} \quad (2)$$

where S indicates 3 digit SIC80 industry and L_{i0} is the employment level that the ONS records to stratify the sample every year. This figure is present both in the ARD and in the "population" (the so called "non selected sample") from which the sample is drawn and makes it possible to compute both L_{i0} and L_{S0}. It also makes it possible to weight the statistics we compute to get aggregate figures.

From equation (2), we need the industry capital stock series K_{St}^{j} over the relevant period.[6] As starting values we use Oulton and O'Mahony (1994) capital stock estimates for 1979. From these numbers we generate the whole aggregate series using the perpetual inventory method at the industry level. The investment series are generated consistently at the establishment and at the three-digit industry level.

The appendix details the method we followed to estimate the capital stock at the establishment level. First, it discusses how to measure investment at the establishment level; then how to aggregate it and finally how to generate the aggregate capital stock series. It also presents a test on the reliability of our estimate of the initial value of the capital stock at the establishment level (the so called "re-entry test"). The "re-entry test", as well as other indirect assessments of the reliability of our capital stock estimates, give positive and reassuring responses. Hence in the next section we present some descriptive statistics on capital stock data we obtained.

2.3. Some descriptive statistics on capital stock data

This section provides descriptive statistics on the estimated capital stock. This is a worthwhile exercise, as capital stock figures at the establishment level, further

[6] Notice that we need the aggregate capital stock series for the whole period and not only for the initial period. This is because our sample is not a balanced one and some establishment will enter it in any period.

disaggregated by asset, are usually not easily available[7]. All tables are in the appendix. Here we sketch some comments. Notice that in this section we focus on capital only; investment figures are presented in the next section.

Median real capital stock (1980 prices) at the establishment level is shown in Table 12. In 1992 it is about 450 thousand pounds in the 20-50 employees class, it goes up to about 1 million pounds in the 50-100 employees class, to 3 million pounds in the 100-500 employees class, to 12 million pounds in the 500-1000 employees class and finally to 35 million pounds above a thousand employees. As expected these values increase with size and over time. It increased of about 50% in all size classes (40% in the smallest one) over the 13 years considered.

If we disaggregate further by output quartiles (Table 13) we see that, within every size class, the median capital stock at the establishment level is increasing with output, while the growth rate over 13 years is only mildly correlated with output.

One would expect a positive correlation between capital (K), output (Y) and employment (L), which is what we find. We compute correlation coefficients among them in Table 1. In the first rows of the table (case O) the positive correlation emerges and it is statistically significant; however, it is quite "low". This is due to the presence of few large outliers in the distributions of $\frac{K_{it}}{Y_{it}}, \frac{K_{it}}{L_{it}}, \frac{Y_{it}}{L_{it}}$ (computed by year and size class), that affect parametric statistics like correlation coefficients. In fact, excluding the first and the last centiles of

[7] We include all establishments (i.e. we consider N=1, see the appendix for details). This because we are quite confident on the estimate of the initial value of the capital stock, and because we want to produce a quite comprehensive set of tables that are not affected by sample selection problems.

the said distributions (case A) all the correlation coefficients increase. If we exclude the first and last 5% of the distributions we obtain even stronger correlations (case B).

Table 1: Correlation coefficients between capital (K), employment (L) and output (Y), by size class

There does not seem to be a correlation between the outliers just mentioned and *n,* where *n* is the distance between the first time the plant is observed and *t*, the current year. In other words, the fact that the initial estimate of K is close or far away from the time the outlier is observed does not seem to matter. This should be obvious in the case of Y/L (both observed values), and it is also clear in the case of K/L: K_0 is estimated proportionally to a function of L. The fact that also K/Y does not assume outlying values closer to K_{i0} might be another indication of a reasonable estimate of K_{i0}. As an example see Figure 1, where the share of outliers in the 1st, 2nd to 5th, 95th to 98th and 99th centiles of the distribution of K/Y are plotted as a function of *n*[8].

Figure 1: outliers of K/Y as a function of n, 100 employees or more.

We now turn to some descriptive statistics on the estimated establishment level capital stock *by asset*. "Plants and machinery" is the largest aggregate within total capital. Its share is increasing from about 55% in 1980 to about 60% in 1992; it is mildly increasing with employment in the establishment. Its median real value (1980 prices) goes from 180 thousand pounds in the 20-50 size class in 1980 to almost 22 million pounds in 1992 in the size class above 1000 employees (Table 14). "Land and buildings" is the second largest aggregate (Table 15): about 40% of total capital; its share is decreasing over time from 40% to about 35%, and it is decreasing more the larger the establishment. Median values at the establishment level go from 140 thousand pounds in the smallest size class in 1980 to 11 million pounds in 1992 in the largest size class; the real value of buildings and land is increasing from 1980 to 1992 by 27% in the 20-50 size class, by 18% in the 1000+ size class. As a comparison, the value of plant and machinery grew by more than 50% in all size classes over the same period. Finally, "vehicles" amount to only about 1% of total capital stock at the plant level; the share is larger the smaller the establishment, and the share is decreasing over time by more than 50% in all size classes (Table 16). Disaggregating further by size class and output quartiles (Table 17), we see that the median share of plant and machinery over total capital stock is mildly increasing with output within each size class; over time – within each size class – it is increasing more the higher the output[9].

3. Investment rate at the establishment level: empirical evidence on non-convexities and irreversibilities.

Traditional theories of investment (of which Chirinko, 1993, presents an exhaustive survey) have used, partly for analytical simplicity, the hypothesis of convex cost of adjustment of the capital stock to its desired level. An implication of such an assumption is that firms will want to avoid large changes in the capital stock and will, therefore, adjust frequently and in small amounts. Recent studies (Doms and Dunne, 1998, Cooper,

[8] It includes the largest size class only (above 100 employees), where sample selection should be least strong.

Haltiwanger and Power, 1999, Nilsen and Schiantarelli, 2000) using US and Norwegian microeconomic data have shown empirical evidence against this implication. In particular, investment seems to occur in large and concentrated episodes. This type of behaviour seems to indicate that non-convex costs of adjustment are a more plausible and realistic alternative (see Rothschild, 1971).

In this section, we characterise the main features of individual investment behaviour in our data. The emphasis will be on those statistics that are directly relevant for judging the importance of non-convex costs. In this respect the evidence we present is very descriptive. We want to establish some stylised facts that should constitute the benchmark of models of investment. The fact that we illustrate are comparable to those documented by Doms and Dunne (1998) for the US, as we will underline when appropriate. It is worth noting immediately that Doms and Dunne (1998) provide what it is likely to be a lower bound of rigidity in the US; in fact they use a balanced panel of establishments, i.e. large establishments only; they analyse total investment, not disaggregated by asset; finally they have net investment, instead of acquisitions and disposals separately. As we will document below, all these characteristics decrease the *observed* rigidity in adjusting capital stock. Every improvement in disaggregating investment (by asset and/or in its gross components) should increase the observed lumpy behaviour of investment decisions at the plant level. The only evidence in this sense that Doms and Dunne (1998) provide is the effect of aggregating the unit of observation further from the plant to the firm, and to the industry level; as expected lumpiness decreases.

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[9] In the largest size class (100 employees or more), the share of plant and machinery increases more with output and

The investment rate is defined for each category of fixed asset as

$$ir_{it}^{j} = \frac{I_{it}^{j}}{K_{i,t-1}^{j}} \tag{3}$$

where I is investment at 1980 prices, K is capital stock at 1980 prices and superscript j stands for each category of fixed asset (plant and machinery, buildings and land, vehicles)[10]. Since we are interested in non-convexities and corner solutions, we avoid smoothing out our investment data by aggregating over different types of investment. Obviously studying separately the different components does not mean that decisions on them are independent of each other (we address this point below). The added value of this procedure is the possibility of comparing results obtained splitting the capital stock by asset to results on total capital, and hence of analysing the effect of aggregating over heterogeneous assets. Furthermore, in equation (3) we can define I as *net* as well as *gross* investment, as we have data on disposals. In what follows we will exploit also this feature of the data.

3.1. Cross-section distribution of investment by asset: looking for non-convexities and irreversibilities

In this section we look at the cross section distribution of investment rates with the aim of providing evidence of non-convexities and irreversibilities.

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growth rates are more diversified by output quintiles. This is obviously the effect of the larger heterogeneity of output and of plant characteristics in general in this wider size class.

[10] Establishments that did not report information in one of the referred categories were removed from the analysis. We removed 7,559 observations involving 504 establishments. This was mainly due to missing values in investment in vehicles or in investment in all three assets at the same time.

Figure 2 plots the distribution of net investment rates by asset and by size class[11]. For Vehicles, as for Buildings and Land, there is a clear spike at zero investment in all size classes. For Plant and Machinery the distribution is smoother; however, a spike at zero is evident when establishments below 100 employees are considered. Notice also the considerable right-skewness of the distribution of investment rates in all assets. In fact, as larger investment rates are considered, the intensity of occurrences declines; still there is a significant share of investment episodes larger than 15% of existing capital stock (spikes). The behaviour for total investment is quite similar to the one described for Plant and Machinery, perhaps not surprisingly, since in our sample the investment accomplished in this category is 77.4% of total investment, We notice, however, that the total is smoother than the individual component. perhaps as an effect of aggregation over heterogeneous assets.

Figure 2: Empirical distribution of investment rates, by size class and asset

In Table 2 to Table 4, we look at the cross section distribution of investment rates in more detail. Notice again that, unlike other studies, as we have separate information on acquisitions and disposals, we can both construct net investment rates (therefore, we can observe negative investment rates) and analyse gross investment flows. In Table 2, we consider five different classes of investment rate: $ir<0$, $ir=0$, $0<ir\leq\delta$, $\delta<ir\leq0.2+\delta$ and $ir>0.2+\delta$. In this table, investment is defined as acquisitions net of disposals. We note the following points:

[11] Minimum and maximum abscissae correspond to 1st and 99th centiles of the *ir* distribution. Notice that the total over size classes is not weighted to represent the population of establishments.

(a) Negative investment rates are interesting because their occurrence gives some evidence on the importance of irreversibility of investment. When we have a negative investment rate, it means that, independently of its magnitude, selling of assets took place. We do not have evidence on which assets are actually disposed of and how much the firm gets for the sale of these assets relates to the book value net of depreciation. However, this evidence is somewhat informative about the importance of irreversibility. If investment was completely irreversible, disposals would not occur, simply because the asset previously bought is firm specific and no other firm is interested in buying it. On the other hand, in the presence of perfect second hand markets, negative investment rates should be normally observed. Therefore, the intensity of occurrence of these negative rates may give a rough idea of the degree of irreversibility of investment previously undertaken.

Table 2 provides a conservative indication of the existence of irreversibility; in principle every time we observe positive disposals – even if net investment is non negative – we have evidence against irreversibility. We can improve this measure exploiting the availability in the ARD of separate information on acquisitions and disposals. Table 3 addresses this point, contrasting positive recorded disposals and negative net investment rates, considering both the number of events and the share of capital stock involved.

(b) Zero investment should be a very unlikely event, unless the cost of adjustment function is non-convex. The frequency of zeros in investment gives us, then, an idea of the presence of non-convexity of the cost of adjustment function; Table 2 considers zero *net* investment rates, while Table 4 contrasts zero acquisitions and zero net investment rates.

(c) When investment rates are below depreciation ($0<ir\leq\delta$) investment does not even replace fully depreciated capital stock. This case could be considered together with the zero investment case, as a small level of maintenance might generate no adjustment costs.

(d) We consider the case in which investment is above replacement but below a 20% net level ($\delta<ir\leq 0.2+\delta$) to be an intermediate case. In the case of large fixed costs (but not linear costs) we should not observe many investment episodes within this range.

(e) Finally we consider large investment episodes those for which investment rates are 20 percentage points above depreciation rate ($ir>0.2+\delta$). These are situations of large or lumpy investment, that is, of an investment spike. As 20% is an arbitrary threshold, we perform the same analysis using a 15% threshold in the second panel of the table. As it can be checked, nothing substantive changes when we use this different threshold.

In what follows we look at each asset separately, as there are obvious disparities in the investment rates' behaviour between asset categories. On this point, notice that investment decisions at the establishment level in each asset are neither independent nor strongly correlated, as Figure 3 shows. The strongest correlation is observed between investment in "buildings and land" and in "plant and machinery"; the coefficient of correlation is about .25 all-over the period. Correlations between investment in the two said assets and in vehicles lays between .15 and .05 and it is decreasing over time[12].

Figure 3: Correlations between investment rates in different assets at the plant level

Considered jointly, Table 2, Table 3 and Table 4 suggest the following observations.

[12] All correlations are significant at the 99.9% level.

For Buildings and Land, the proportion of zero investment is very high (58%, and a bit higher considering zero acquisitions), highlighting the importance of non-convex adjustment costs for this particular asset. At the same time, the fact that 61% of total investment is accounted for the mere 5.5% of spike investment episodes emphasises the lumpy character of investment in this category of asset. Irreversibility seems not absolute: 5.4% of negative investment rates; however, considering positive disposals the number of episodes almost doubles. As easily expected, the amount of disposals over the stock of capital is on average lower than the average negative investment rate; figures are respectively 22% and 17% (this feature is common to all assets). More episodes of smaller disposals are clear evidence of less irreversibility than expected considering only net investment rates. Summing up, investment in Buildings and Land faces non convex adjustment costs that are revealed by infrequency of adjustment and by investment in large chunks; this kind of investment does not seem strongly irreversible.

Vehicles is the only category for which we observe a substantial number of negative investment rates (13.7%). Furthermore, the number of establishments reporting positive disposals in Table 3 is five times that of establishments with negative investment rates: the number of observations with positive disposals is above 100,000 (out of about 180,000 observations). Irreversibility seems to be much less of an issue here, perhaps not surprisingly since, with some exceptions, a vehicle is hardly a firm or industry specific asset. The number of zeros and spikes is similar (25%); however the number of zeros increases by one fifth considering zero acquisitions. The lumpy character of investment is still present, with said 25% of investment rates above the spike threshold accounting for 58% of total investment in Vehicles. Compared to investment in Buildings and Land, investment in Vehicles is quite easily reversible; the adjustment cost function appears to be non convex, although investment in Vehicles is less infrequent with respect to investment in Buildings and Land, and the same share of total investment accounted for by spikes (about 60%) is due to 25% of plants instead of 5%.

The behaviour of Plant and Machinery differs substantially from that observed for Vehicles and for Buildings and Land. The occurrence of very few zeros (2.3%, although it increases by one seventh considering zero acquisitions); the number of disposals ten times higher than the number of negative investment rates; a majority of values between the zero investment and the spike threshold (87% of events accounting for 75% of total investment in Plant and Machinery) and 7% of spikes accounting for little more than one fourth of total investment in Plant and Machinery leads to the conclusion that the presence of fixed adjustment costs might have, for this category, a limited role.

Table 2: Distribution of investment rates

Table 3: Positive gross disposals vs. negative net investment rate.

Table 4: Zero gross acquisitions vs. zero net investment rate.

It is clear that different assets show quite different behaviours. Had we analysed only an aggregate "total" category, we would have concluded that capital is a fairly flexible asset, as only 5.4% of all observations are what we define 'spikes', 6.7% of observations register negative investment rates (and ten times higher when considering positive disposals), and only 1.2% of all observations register zero investment (that increases only by one fourth considering zero acquisitions). The very lumpy behaviour of Buildings and Land, and to a more limited extent also of Vehicles, is not observable any more. Given the large effect one gets aggregating assets, it is natural to ask whether the relative flexibility of Plant and Machinery is real or reflects its much greater heterogeneity compared to the other two assets. Unfortunately, not even our very rich data-set can answer this question directly. To shed more light on this point, however, we may look at simultaneous investment and disinvestments *in the same asset* at plant level: more heterogeneous assets should have more of these episodes. Table 5 shows the contemporaneous occurrence of acquisitions and disposals at the plant level, contrasting investment in Plant and Machinery and investment in Buildings and Land. If we consider Plant and Machinery we notice that contemporaneous acquisitions and disposals are quite a common event: 23.3% of small plants, 34.3% of medium and 57.7% of large plants buy and sell capital assets in the same year[13]. These percentages drop respectively at 3.5%, 5.2% and 10.8% considering Land and Buildings[14]. As expected, the more heterogeneous asset experiences more episodes of contemporaneous acquisitions and disposals at the plant level.

Table 5: Contemporaneous occurrence of acquisitions and disposals at the plant level

With some caution, we can compare our results with Doms and Dunne's (1998) on the US manufacturing plants. Considering *net* investment in *all* assets from Table 2 is what allows us to get figures that are comparable to Doms and Dunne. However, we have to remember that our panel includes small plants while Doms and Dunne's (1998) balanced

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[13] Nothing substantial changes if we exclude "work of capital nature by own staff" from the definition of acquisitions of Plant and Machinery.

[14] If we exclude from the definition of acquisitions maintenance investment, i.e. acquisitions below the depreciation rate, we obtain lower percentages of contemporaneous acquisitions and disposals, but the quality of the results is unchanged.

panel excludes them. They record 51% of plants investing less than 2.5% and making up 19% of total investment. We find that 63% of establishments invest less than 6% (which is the average depreciation rate). These account for 26.8% of total investment.[15]. Furthermore, Doms and Dunne (1998) find that 11% of plants have an investment rate above 25% and that they generate 25% of total investment. Our figures indicate that 5.4% of establishments have investment rates above 26% (20% plus depreciation) and they account for 24.6% of total investment. Therefore, we conclude that in terms of percentage of firms investing a substantial amount and in terms of how much aggregate investment is accounted for by those firms, the US and the UK are remarkably similar. There are some differences, instead, in terms of the percentage of firms with zero or near zero investment. However, the higher percentage of plants with near zero investment and the higher share of investment that we obtain with respect to the US can be easily explained by the higher threshold we use and by the smaller average plant size in our dataset. Even in this respect, therefore the comparison provides some evidence of a comparable degree of rigidity in capital investment in the two countries. At the same time, our numbers indicate both the importance (and the different behaviour) of small firms and the importance of distinguishing among different assets.

3.2. Investment rate by size of the establishment

As we expect some substantial differences between small and large firms in terms of the importance of costs of adjustment, it is worth focusing on the relation between the lumpy

[15] As mentioned, our depreciation rate is a weighted average of the various asset's depreciation rates: 7% for plant and machinery, 2.91% for "land and buildings" and 28.1% for "vehicles".

behaviour of investment and plant size. We expect smaller plants to face higher rigidity in adjusting their capital stock and therefore exhibit ‘lumpier’ investment patterns, as also found by Doms and Dunne (1998) for the US They obtain this result in a multivariate analysis of investment spikes. Our analysis is going to be slightly different, as we simply repeat the analysis on the frequency of zeros and spikes by plant size.

The information concerning the frequency of zeros and spikes by size of establishment is detailed in Table 6 for each type of asset, taking averages over the whole period and considering *net* investment. The establishment size affects the number of zeros observed both for Buildings and Land and Vehicles. For both assets, there is a decrease in the number of zeros as bigger establishments are considered. The behaviour of Plant and Machinery does not seem to change with the establishment size; the percentage of zero investment stays constantly low.

The number of spikes decreases with establishment size only in the Vehicles category. This behaviour may lead to the conclusion that given a smaller number of vehicles in smaller plants, it is more likely that the price of one vehicle is above 20% of the value of the existing stock of vehicles.

As expected the share of total investment represented by spikes is decreasing in establishment size. However, the range of variation is very different by asset. For buildings and land it goes from 100% in very small plants to 6.7% in very large ones; for vehicles it decreases from almost 100% to 50% moving from establishment below 50 employees to establishment above the 10,000 employees threshold; finally for plant and machinery it decreases from 50% to 20%.

Table 6: Distribution of zeros and spikes by size of establishment

Disposals by size of the establishment shed more light on the importance of irreversibility in small plants. Table 7 shows a clear regularity: the percentage of plants recording zero disposals decreases as establishment size increases; i.e. disposals are more frequent events the larger the plant. Furthermore, also median disposal relative to capital stock, when disposal is positive, decreases as establishment size increases[16]; i.e. small plants dispose capital less frequently, but when they actually do it they dispose larger shares of their capital stock. This is a clear indication of non-convex costs of dis-investment among small plants. It is less so among larger plants. This conclusion holds for all assets: buildings and land, vehicles as well as plant and machinery.

Table 7: disposals by size

3.3. Spikes and lags: looking for infrequency of adjustment

If investment spikes (and zeros) occur because of the presence of non convex costs of adjustment, one should find that investment spikes are relatively infrequent (unless they 'spill over' different years). In this subsection we look at the frequency for each plant, of investment spikes. To perform this analysis we select only establishments with more than 100 employees (as they are more likely to be sampled every year) and we follow those establishments that have a spike between 1980 and 1985. We classify them in "0 years" if the next spike occurs in the year after the first spike is observed, in "1 year" if the next spike occurs with one year interval, and so on. "No more spikes till end" means that the establishment stays in the sample till the end of the observation period and doesn't experience another spike. "No more spikes till out of sample" means that the

[16] It is only obvious to notice that the scale matters: the absolute value of median real disposals is increasing with plant size.

establishment leaves the sample before the end of the observation period and doesn't experience another spike.

The number of establishments with a spike in 1980-85 is 1866 for Buildings and Land, 2903 for Plant and Machinery and 8203 for Vehicles. The average number of years in the sample for each establishment is 4.8.

As large investment episodes are likely to spill over different years, one would expect to see many consecutive spikes. In fact Table 8 reports in "0 years" 19.2% of plants for Buildings and Land, 25.5% for Plant and Machinery and 23.1% for Vehicles. Besides the one-year lag, however, spikes seem to be separated by relatively large intervals. For about half of the establishments in our sample, we observe only one spike.

To sum up, spikes seem to occur either consecutively or very wide apart. This is consistent with the presence of non-convex costs. It is also consistent with the evidence in Doms and Dunne (1998), in fact they find that "on average, half of a plant's total investment over the 1973-1988 period was performed in just three years"

Table 8: Spikes and Lags

3.4. Investment over time and the business cycle

From an aggregate point of view, it is well known that investment is one of the most volatile components of GDP. It is therefore interesting to check how this volatility is originated at the micro level. In this section we consider the time series properties of some of the statistics we have been considering. Of course, our analysis is limited by the time series length of our sample. We analyse the behaviour of zeroes and spikes, as well as of the first three moments of the cross section distribution of investment rates. For zeros and spikes, as well as for mean investment at the plant level our expectiations are quite clear cut: we expect pro-cyclical mean and spikes, and anti-cyclical zeros. Doms and Dunne (1998) find pro-cyclical spikes in the US.

The prediction about the cyclicality of the variance and skewness of the cross sectional distribution of investment rates is a bit less obvious. If the behaviour of spikes drives the result then we should expect pro-cyclical variance and skewness. On the other hand, Doms and Dunne (1998) find that in the US, investment is more concentrated when the cycle is high; this should imply anti-cyclical variance and skewness. This result would be consistent with non-convex adjustment costs: high demand would trigger the adjustment decision for many plants at the same time.

If we consider the profile of total net investment over time we notice (Figure 4) that in recession years the spike at zero is more marked among establishments below 100 employees; this is less true above that threshold.

Figure 4: Empirical distribution of total net investment rates, by size class and year

The profile over time of zeros and spikes by asset is plotted in Figure 5. The first thing to notice is that the two lines mirror each other quite well, i.e. when the number of spikes increases the number of zeros decreases, and vice-versa. Zeros and spikes in Plant and Machinery and in Vehicles follow roughly the same time profile, with anti-cyclical zeros and pro-cyclical spikes as expected. Buildings and Land show pro-cyclical spikes and anti-cyclical zeros as well, although the number of zeros stays high for a longer period after the recession of early ‘80s.

Figure 5: Zeros and spikes over time

Spikes account for a large share of total investment, between 30% and 50%, depending on the asset considered (see also Table 2). Figure 6 confirms this and shows that for all assets the share of total aggregate investment generated by investment spikes is pro-cyclical. It is more mildly pro-cyclical and lower for plant and machinery (always below 30%), while it is higher for buildings and land (about 40%) and for vehicles (about 50%) and more strongly pro-cyclical (it increases by 10 percentage points in the late '80s boom). Again this is consistent with a lumpy behaviour of investment, more important for vehicles and buildings and land, less so for plant and machinery.

Figure 6: Share (%) of total aggregate investment by asset generated by investment spikes at the establishment level; and aggregate output (,000Billion)

In Figure 7 we plot median values of investment spikes at the plant level, by asset. They do not vary much over time. If anything, they seem to be pro-cyclical, with the exception of median investment in vehicles, that increases steadily over time.

Figure 7: Median investment rate spike at the plant level, by asset; and aggregate output (,000Billion)

In Figure 8 we plot the first three moments of the cross section distribution of investment rate for each category of asset for each year. Panel (a) shows that mean investment rate is strongly pro-cyclical, and that the turning point toward the early '90s recession is one year anticipated with respect to aggregate output for all assets. Panel (b) and (c) show that the coefficient of variation (M2) and of skewness (M3) are clearly anti-cyclical (they are plotted with mean investment). This is consistent with Doms and Dunne's (1998) result we mentioned above, and with the existence of non-convex costs of adjustment.

Figure 8: Moments of the cross section distribution of investment rate, by asset and year

4. Conclusions

In this paper we have presented descriptive evidence of establishment level investment decisions in the UK. For such a purpose we have used a data set which includes a large number of establishments over the period 1980 to 1992. In the first part of the paper we described the construction of measures of the capital stock and showed that our approach yields sensible results. We then moved on to assess the importance of non-convex costs of adjustments. We document the occurrence of spikes and zeros in establishment level investment rates. For this purpose our data set is particularly interesting as it reports investment by three assets (Buildings and Land, Plant and Machinery, Vehicles) and it also contains separate evidence on disposals.

Overall the evidence we present is consistent with that coming from the US. As in the US, a large fraction of aggregate investment is accounted for by few establishments that are investing a lot. Furthermore, for each establishment, a large fraction of its investment activity over a long horizon is accounted for by a few large episodes. I.e., in all assets spikes seem to occur either consecutively or very wide apart. Still consistently with US evidence, small plants seem to face non convex-adjustment costs in all assets to a larger extent with respect to larger establishments. Finally, cyclical behaviour of investment, and of the first three moments of its cross-section distribution, are consistent with the existence of non-convex adjustment costs.

In particular, we find that investment in Buildings and Land faces non convex adjustment costs that are revealed by infrequency of adjustment and by investment in large chunks; this kind of investment does not seem strongly irreversible. Compared to investment in Buildings and Land, investment in Vehicles is quite easily reversible; the adjustment cost function appears to be non convex, although investment in Vehicles is less infrequent with respect to investment in Buildings and Land, and the same share of total investment accounted for by spikes is due to five times the number of plants. The behaviour of Plant and Machinery differs substantially from that observed for Vehicles and for Buildings and Land. The occurrence of very few zeros, the high number of disposals, a majority of values between the zero investment and the spike threshold and only few spikes leads to the conclusion that the presence of fixed adjustment costs might have, for this category, a limited role.

It is clear that different assets show quite a different behaviour. Had we analysed only an aggregate “total” category, we would have concluded that capital is a fairly flexible asset; the very lumpy behaviour of Buildings and Land, and to a more limited extent also of Vehicles, would not have been observable any more. Given the large effect one gets aggregating assets, it is natural to suspect that the relative flexibility of Plant and Machinery reflects its much greater heterogeneity compared to the other two assets. All this points to the importance of detailed and highly disaggregate data on investment to be able to perform sound micro-econometric analysis on investment decisions.

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6. Appendix

As anticipated in section 2, this appendix details the method we followed to estimate the capital stock at the establishment level. First, we discuss how to measure investment at the establishment level; then we explain how we aggregate it and finally how we generate the aggregate capital stock series. We also perform a test on the reliability of our estimate of the initial value of the capital stock at the establishment level (the so called “re-entry test”). At the end of the appendix some descriptive tables on capital stock data are reported. They are discussed in section 2.3.

A foreword about the panel dimension of the data is necessary. As already mentioned, the ARD is not a panel but a series of cross sections independently drawn every year. The stratified nature of the sample implies that only establishments above 100 employees are included in the archive with probability one every year, provided that they do not shrink below the threshold. It may happen that the same establishment enters and exits the sample several times over the observation period[17]. Hence, selecting establishments that are observed for more than *N* consecutive years, where *N*>1, imposes a strong and non-random selection of the sample. Selection problems are likely to be stronger the smaller the establishment and the longer *N*. In fact only establishments that are consistently

[17] If there is more than one continuous series of observations over time for the same establishment, in the process of estimating the capital stock, we consider them to be as two different establishments. That is, we estimate the initial level of capital stock from scratch every time the establishment re-enters the archive. This procedure allows us to

"large" over time match the criteria to be included in the sample for several years in a row. In deciding the length of *N*, we have to assess the relative loss of representative-ness of the sample against the increased relevance of the guess about the initial value of the capital stock. Clearly, the two biases cannot be separately identified and estimated; however one can try to assess their effect in an indirect way, as we do below. Because of the descriptive purposes of this paper, most of the time N=1 will be appropriate. This avoids sample selection and makes the statistics presented representative of the population of manufacturing plants in the UK. Of course the reliability of the estimated K_{i0}^{j} is crucial.

6.1. Investment series and leased assets

Net investment by asset is defined as acquisitions (A) minus disposals (D), including assets that are not yet in production (*nyip*). In "plant and machinery" we include "own production of capital inputs".

$$I_{it}^{j} = A_{it}^{j} + Anyip_{it}^{j} - D_{it}^{j} - Dnyip_{it}^{j} \tag{4}$$

There is a problem involving leased assets[18]. Before 1988, questions about investment expenditure were supposed to include both purchased and leased assets. From 1988 to 1991, although nothing was supposed to change in the definition of these questions, separate questions were asked about leased assets. After this change, it becomes clear that leased assets were under-estimated before 1988 and that the extent of under-reporting was reduced by the change in the questionnaire. Figure 9 shows the point. We take the Blue Book investment series as our benchmark, because it includes leased assets all over

minimize the loss of observations. We will exploit this feature of the estimation procedure in the "re-entry test" below, to check the quality of our initial capital stock estimates.

the period considered[19]. We plot (i) the Blue Book nominal investment series, (ii) the ARD nominal investment series not corrected, (iii) the "ARD minus leased assets" nominal investment series; all of them aggregate at the "total manufacturing, all assets" level[20]. Notice that the first and the third line lay *parallel* all over the period considered, indicating a roughly constant wedge represented by leased assets; furthermore, the fact that there seems to be no break in the "ARD minus leased assets" nominal investment series around 1988 indicates that almost all leased assets were not reported before 1988 (as they were totally excluded after that date). The non corrected ARD nominal investment series is by definition identical to the third line up to 1987, but it becomes steeper than the other two lines and moves from the lower to the upper line after 1988. The break in 1988 and the fact that it lays close to the first line afterward, clearly indicates that ARD investment amount excluded most of leased assets up to 1988, and included them thereafter.

Figure 9: Nominal investment series, all assets. Manufacturing, m£

Summing up, before 1988 the reported amount of investment seems to underreport leased assets; after that date it is not so any more. However, we know the amount of leased assets at the establishment level from 1988 to 1991. Two options are available: either we exclude leasing L_{it}^{j} from our definition of investment after 1987, or we estimate establishment level leased assets between 1980 and 1987. We follow the first option[21]:

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[18] The problem does not involve Buildings and Land.
[19] In Books published after 1991.
[20] Aggregation method is discussed below.
[21] Summarising, we do the following. We keep investment as they are reported in the period 1980-1987, assuming that they exclude leased assets. We define investment subtracting leased assets from acquisitions in 1988-1991. We reduce proportionally investment in 1992, using average percentage of leased assets over acquisitions, by asset, in the establishment. The average percentage is computed using the available years in the period 1988-1991 in which we observe the establishment, if it invests. If we never observe it in that period and/or if it never has a positive amount of

$$I_{it}^{j} = A_{it}^{j} + Anyip_{it}^{j} - D_{it}^{j} - Dnyip_{it}^{j} - L_{it}^{j}, \text{ if } t>1987 \tag{5}$$

The reason for this choice is that by doing so we have consistent series over time, and because we consider almost impossible to estimate a reliable value for leased assets at the *establishment* level for each year before 1988. However, one should be aware of the fact that we exclude a significant part of investment from our definition, that we could add *after 1988*[22]. This is not a severe problem for us, as the purpose of this paper is to study adjustment costs; leasing might have different adjustment costs or involve different investment decisions, because it is a financed purchase, i.e. the financing plan and the asset are jointly chosen and purchased. On the other hand, who estimates - for example - production functions with these data should be aware of the fact that some of the capital inputs are not fully measured in the ARD up to 1988.

Having estimated investment at the establishment level using equation (4) up to 1987 and equation (5) thereafter, aggregate investment series is computed grossing up the establishment level figures[23].

To check whether our computations produce a reasonable estimate of the aggregate nominal investment series that are published, we compared them to the aggregate nominal investment series published by ONS in the Report on the Census of Production (Figure 10). Our estimation method seems well consistent with the Census' official one when we do not exclude leased assets; the two series are very close, and ours is slightly below the official one because we only include private companies. The difference is

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acquisitions, we use the average percentage of leased assets over acquisitions, by asset, in the 3-digit industry the establishment belongs to.

[22] Moreover, Oulton and O'Mahony capital stock estimates includes leased assets, so we will have to adjust for it.

evident after 1988, due to the different treatment of leasing. Notice also that the estimated investment figures for 1992 are not totally reliable, as it seems that in 1992 there is a break in the investment series that excludes leasing; the same break is not observed in the other series[24]. This supports our choice not to estimate leasing figures at the establishment level for the years before 1988.

Figure 10 Comparison between Official Census nominal Investment series and investment series computed from ARD (including and excluding leasing).

6.2. Aggregate real capital stock series

The starting values for the aggregate capital stock series are those published by Oulton and O'Mahony (1994): three-digit capital stock estimates for 1979 at 1980 prices, converted from three-digit SIC68 to three-digit SIC80.

As Oulton and O'Mahony's figures include leasing, we reduce their estimates proportionally using the average percentage of leased assets over total net investment, by asset, in the three-digit industry. The average percentage is computed using the period 1988-1991 in which expenditure in leased assets is recorded in the ARD, assuming that it has not changed significantly over time. The fact that in Figure 9 the wedge between the first and the third line (i.e. leased assets) was roughly constant over time makes this assumption a bit less heroic. Our procedure induces a 6.6% reduction in the *total* initial capital stock.

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[23] The weight is the inverse of the ratio of employment in the ARD and in the population, in the cell defined by 3 digit industry, size class and year.

[24] This matters in the micro analysis, not in the aggregate investment series. It might just imply that the 1992 estimated aggregate capital stock value is not totally reliable, but it is not used to estimate establishments' initial capital anyway. In fact establishments that enter the dataset in 1992 use a share of 1991 aggregate capital stock.

From the obtained values of capital stock in 1980 we use the PIM as it is done at the establishment level (equation (1)). At the establishment and at the aggregate level we use the same deflators and the same depreciation rates.

Depreciation rates are those used by Oulton and O'Mahony (1994) for "land and buildings" (0.0291) and "vehicles" (0.281). For "plants and machinery", on the other hand, we assume a depreciation rate of 0.07. Oulton and O'Mahony (1994) report five depreciation values for five categories of plants and machinery; the aggregate depreciation rate obtained as a weighted average of these five values is around 11%. We chose a lower value for two reasons. First, Oulton and O'Mahony's depreciation rate already takes into account disposals, while we are explicitly subtracting them from our investment series. Second, and more importantly for plant and machinery, with a depreciation rate of 11%, the aggregate real capital stock series would be decreasing dramatically between 1980 and 1984, and this is not credible. A depreciation rate of 7%, instead, does not cause the aggregate series of the capital stock to decline in the first years of the sample.

Figure 11 shows capital stock series at the "total manufacturing, all assets" level. It compares the capital stock series obtained as described above (Lk) with three other series. They all start from the original Oulton and O'Mahony 1979 value of capital stock (not reduced); then they use different investment series: aggregate ARD including leasing after 1988 (Ak), official Census of production (Ck), Blue Book (Bk) investment series (it always includes leasing). PIM is computed in the same way for every series. The estimated capital stock is roughly constant up to 1984, it increases mildly afterward.

Notice the steep rise after 1988 in the two capital stock series (Ak and Ck) that do not correct for the different treatment of leased assets in the two sub-periods.

Figure 11: Real capital stock series

6.3. A "re-entry test" on the estimated initial capital stock at the establishment level

To test whether the estimate of the initial value of the capital stock at the establishment level is reasonable we do what we label "re-entry test". We select almost 2000 establishments that exit and re-enter the ARD dataset at least once, that have at least 5 consecutive observations in the first spell and that re-enter after a maximum of three years (i.e. we allow for one of two years gap only). Then we *extrapolate linearly* the first series of capital stock values, up to the first year of the second series, and then compare the extrapolated and the estimated capital of the first year of the second series[25]. Table 9 shows the number of establishments involved in the test. It also shows the share of outliers, i.e. the share of involved establishments that *in the re-entry year* were in the 5% tails of the capital per unit of output K/Y distribution (computed in the complete dataset by size class and year). Notice that all shares of outliers are about or below 10%, i.e. the value indicating that these observations are not different from the others, although there is some variability over time.

Table 10 shows the median value of the relative difference between estimated and extrapolated initial capital stock: (Kext-Kest)/Kest. Even if the exercise is quite simple,

[25] As we said, every time the establishment re-enters the ARD we estimate the initial value of capital from scratch, as if it was a new establishment.

based only on a linear extrapolation, median differences are reasonably small: 0.004 in the 20-50 size class, -0.023 in the 50-100 size class, -0.101 in the 100+ size class, although there is some variability over time.

The outcome of this “re-entry test”, although performed on a quite selected sub-sample, supports the claim that our initial capital stock estimates are reliable and quite satisfactory.

Table 9: Establishments in the “re-enter test”, by size class

Table 10: Median distance between extrapolated and estimated initial capital

6.4. Tables

Table 11: Number of establishments in the dataset, by year and size class

Table 12: median capital stock at the establishment level, by size class

Table 13: median capital stock at the establishment level, by output quartiles and size class

Table 14: Median plant and machinery capital stock at the establishment level, by year and size class

Table 15: Median buildings and land capital stock at the establishment level, by year and size class

Table 16: Median vehicles capital stock at the establishment level, by year and size class

Table 17: Median share of plant and machinery over total capital stock at the establishment level, by year, size class and output quartiles

7. Figures

Figure 1: outliers of K/Y as a function of n,

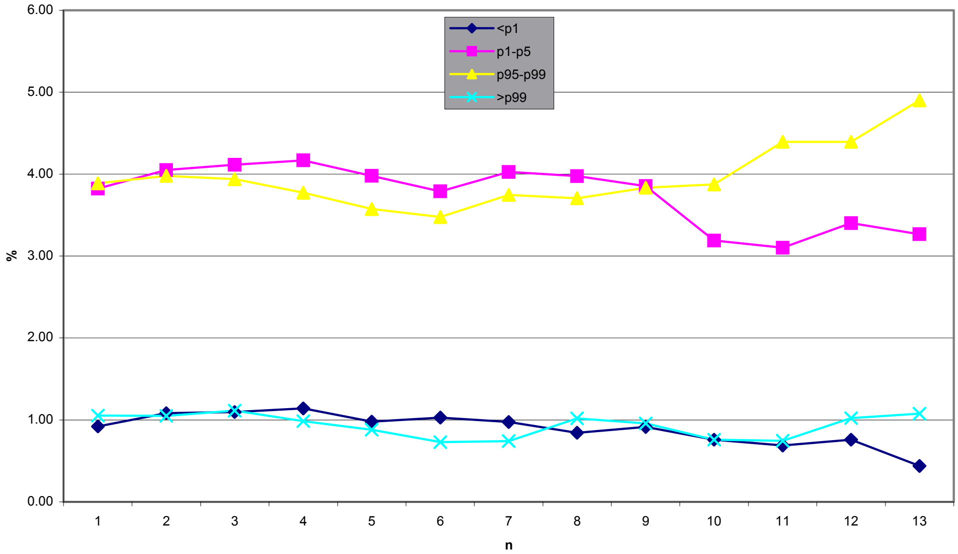
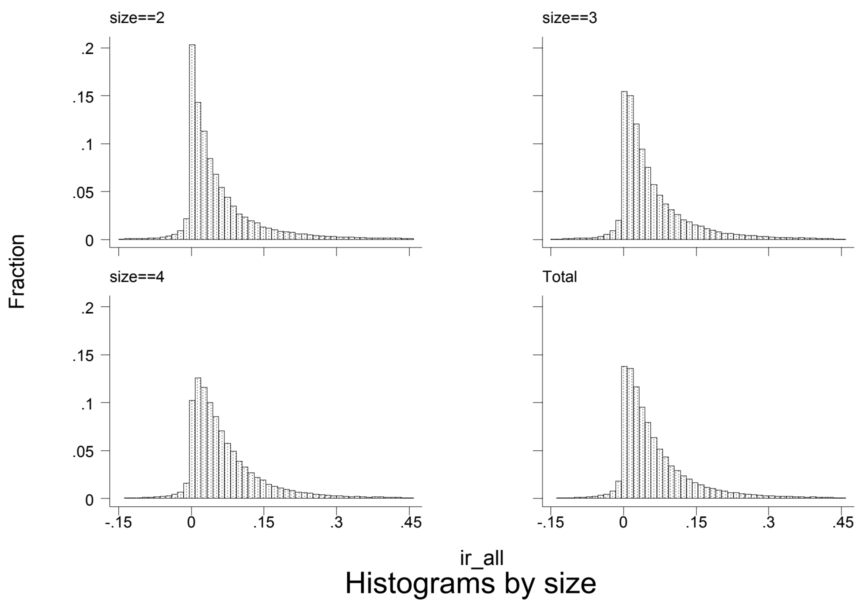
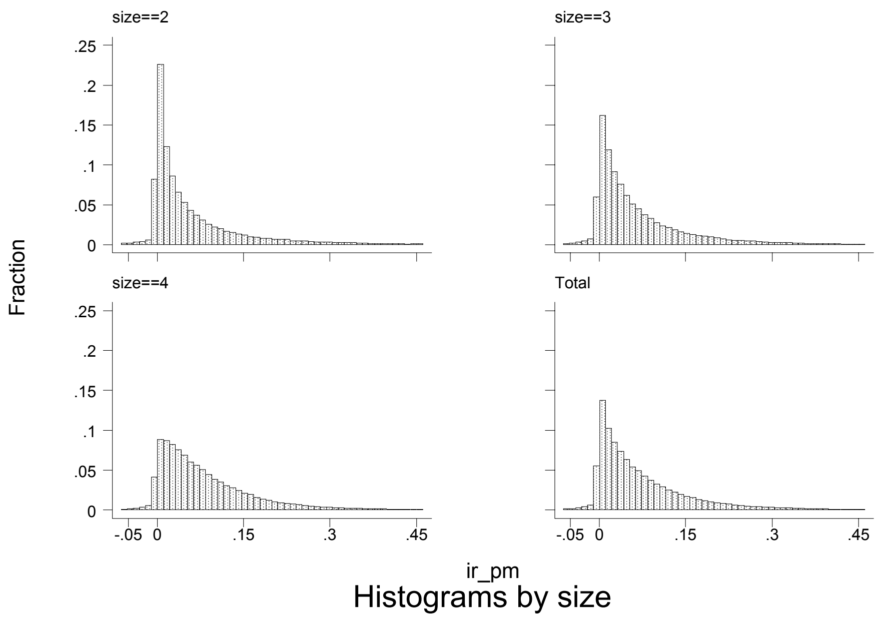


Figure 2: Empirical distribution of investment rates

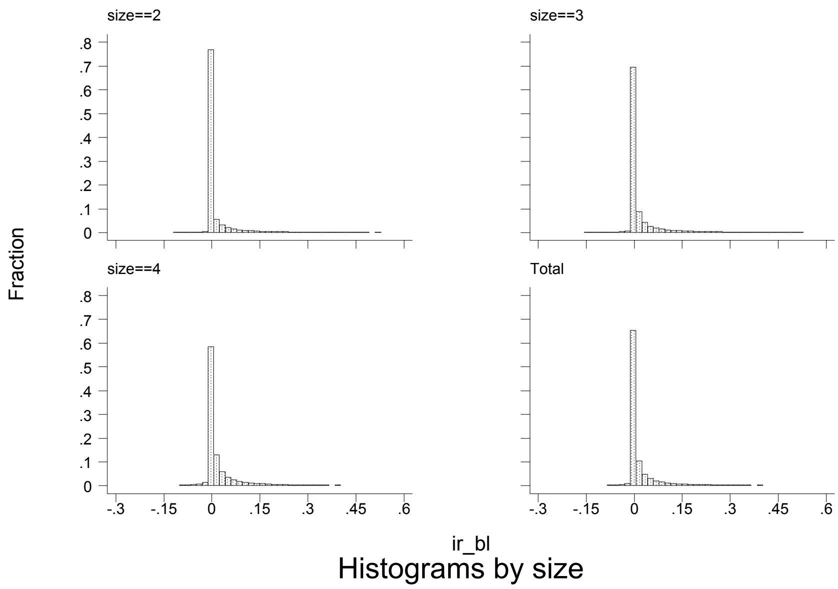
a) total investment (size 2: 20-50, 3: 50-100, 4: 100+)



b) plant and machinery



c) land and buildings



d) vehicles

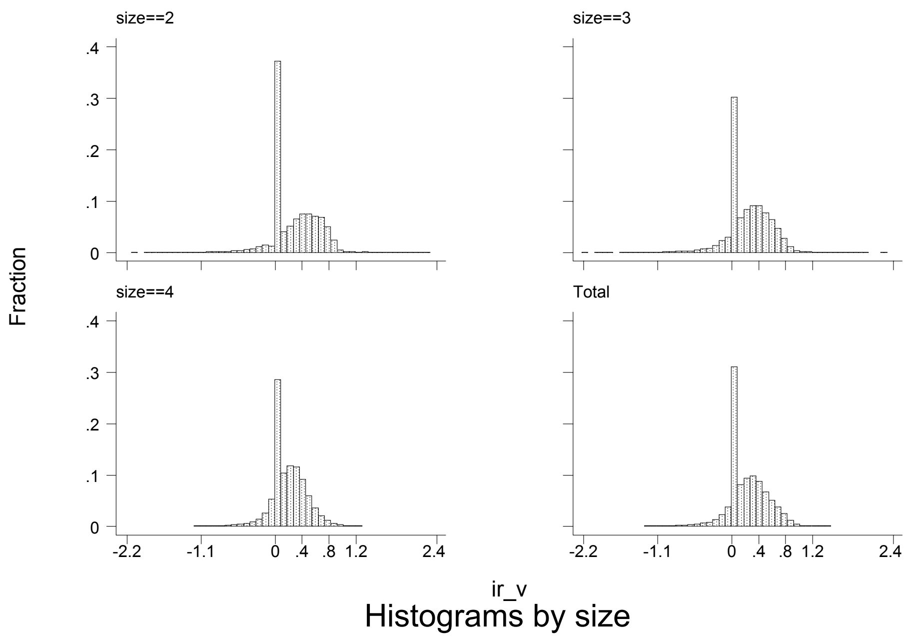


Figure 3: Correlations between investment rates in different assets at the plant level

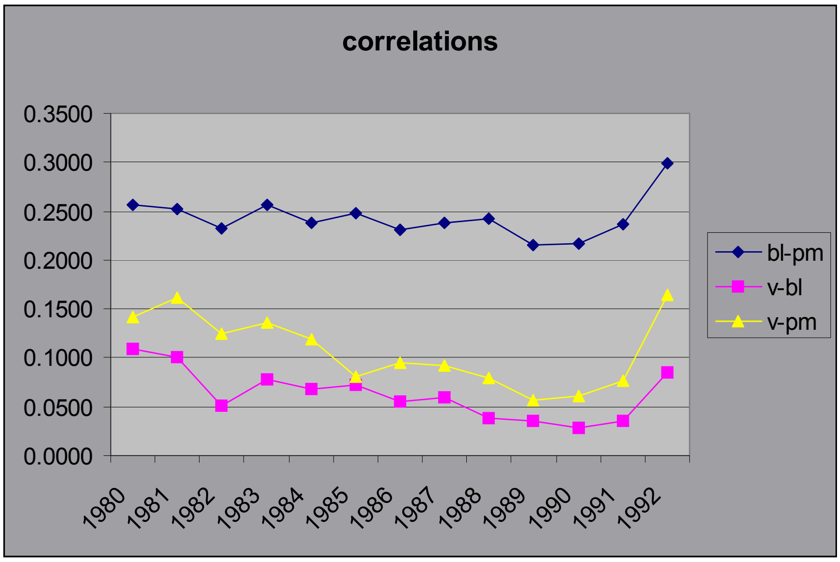
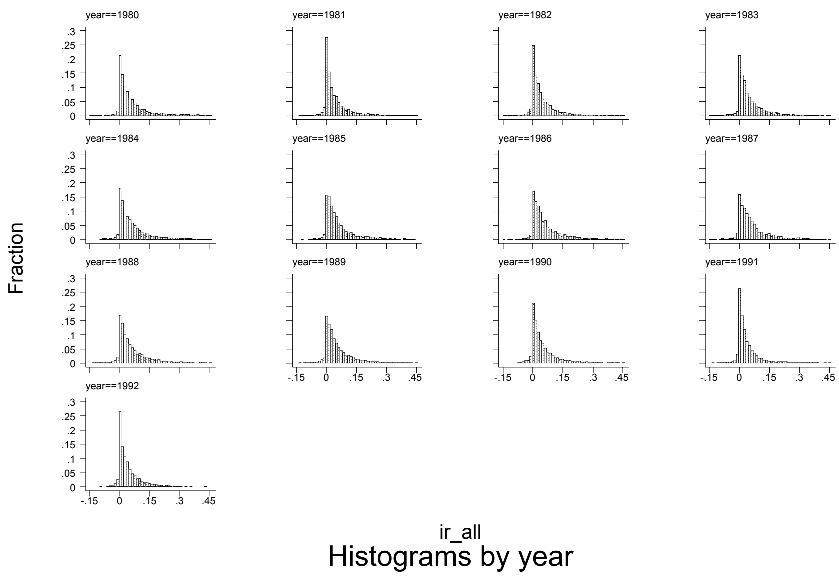
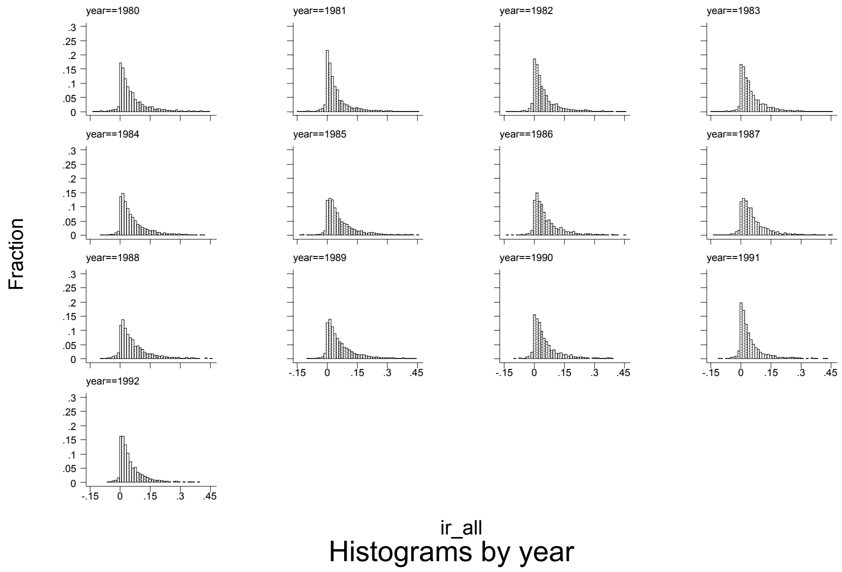


Figure 4: Empirical distribution of total net investment rates, by size class and year

a) 20-50 employees



b) 50-100 employees



c) >100 employees

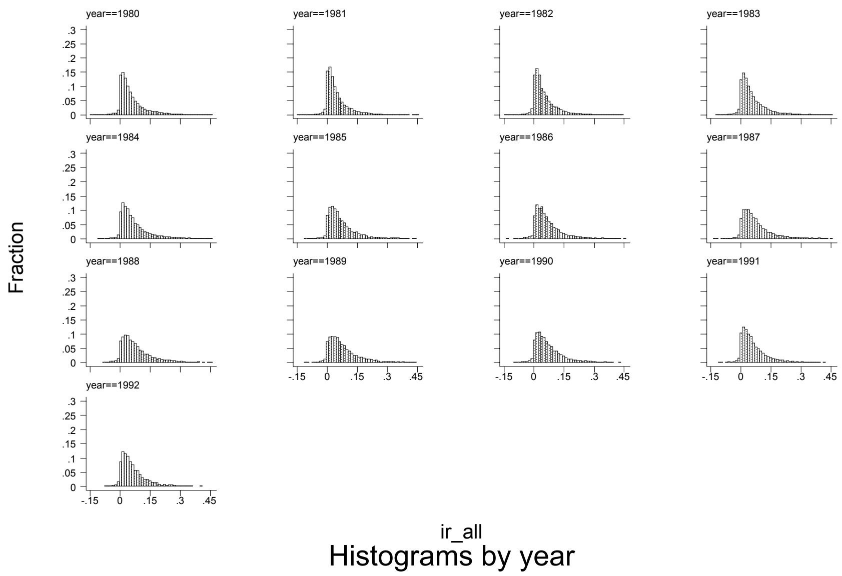
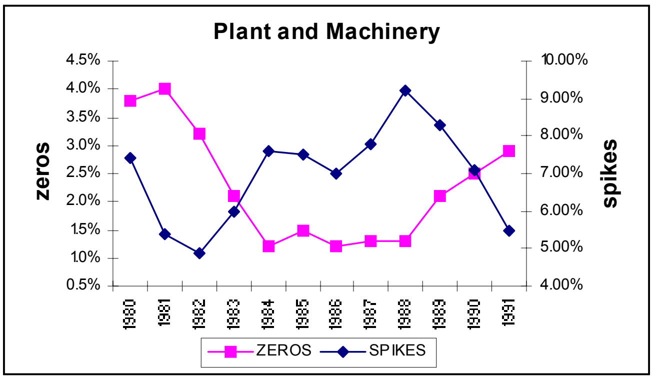
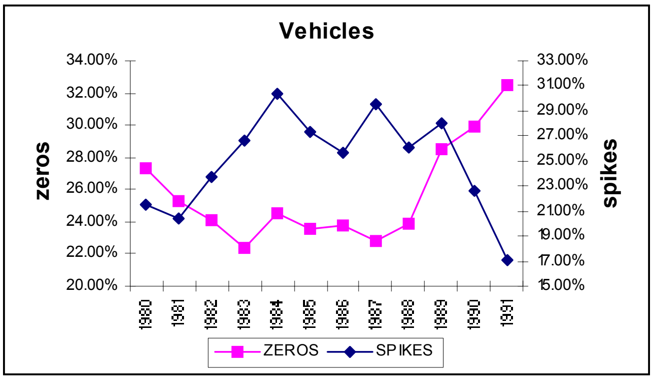


Figure 5: Zeros and spikes over time





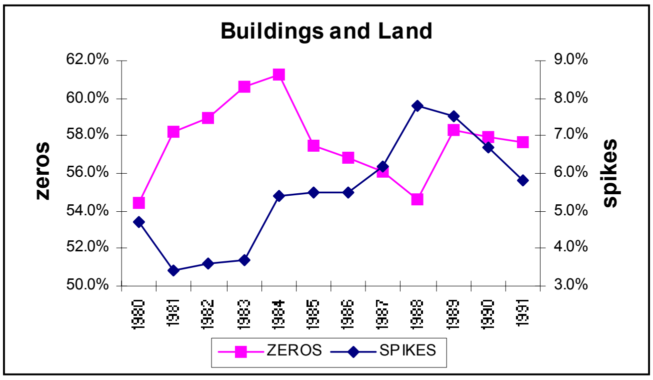


Figure 6: Share (%) of total aggregate investment by asset generated by investment spikes at the establishment level

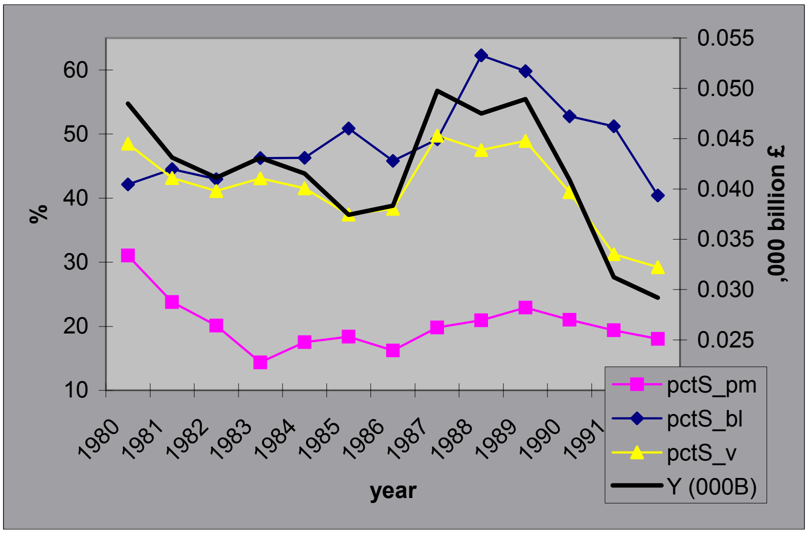
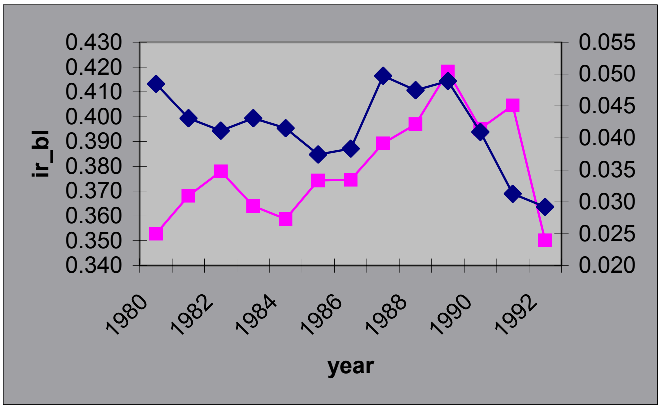
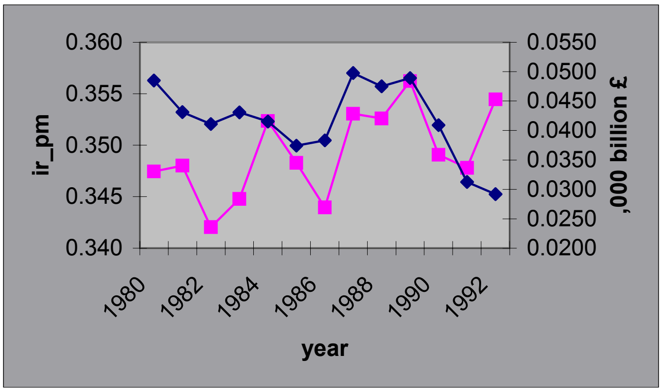


Figure 7: Median investment rate spike at the plant level, by asset





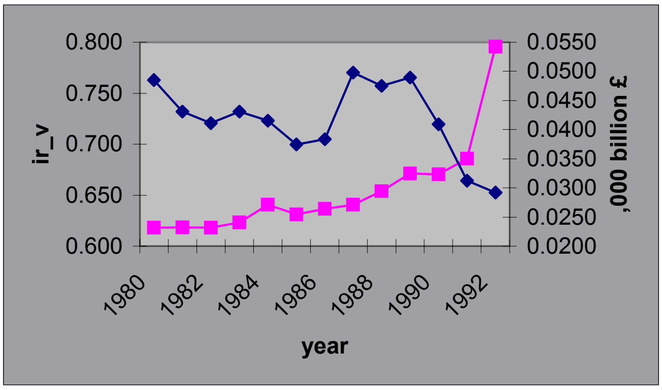
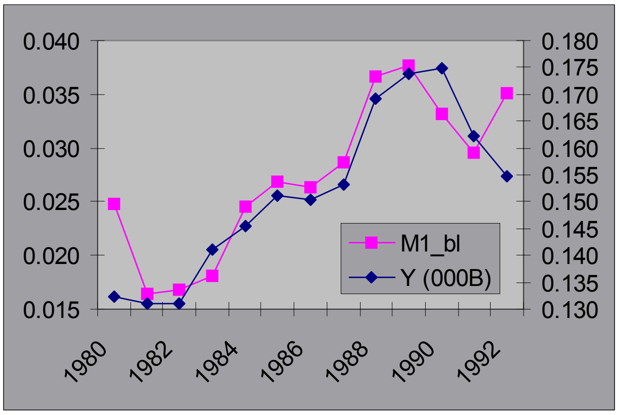
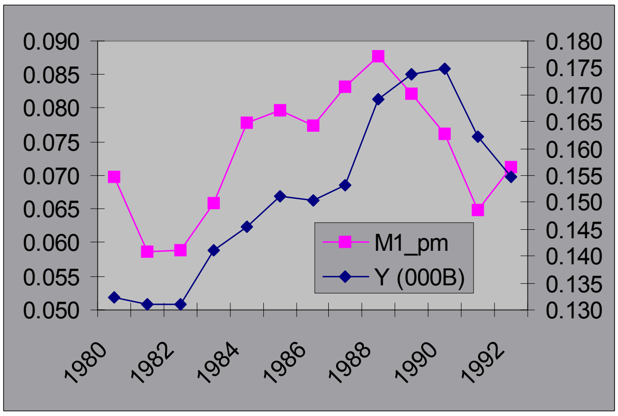
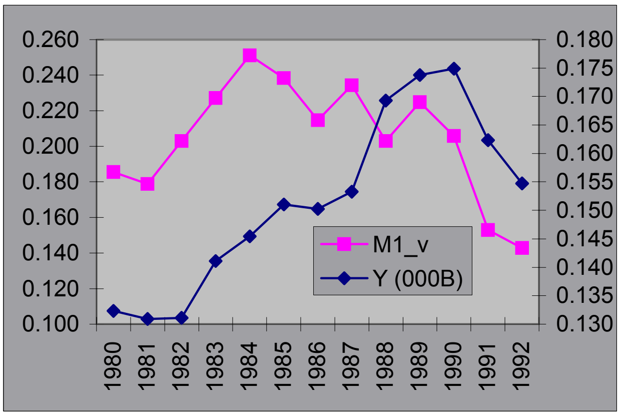


Figure 8: Moments of the cross section distribution of investment rate

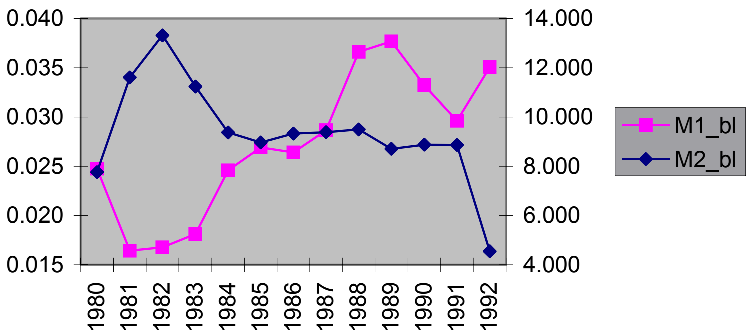
a)mean investment rate by asset and total real output

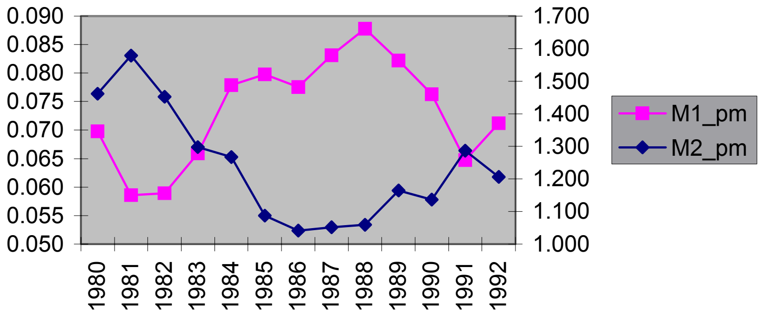


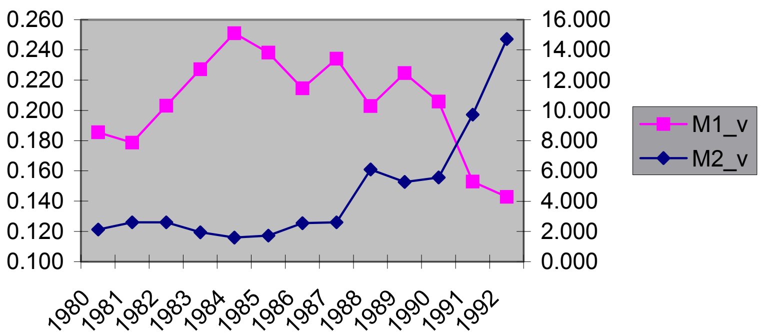




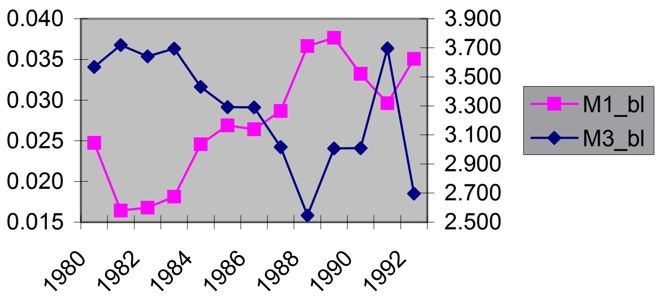
b) Mean investment rate and M2, by asset

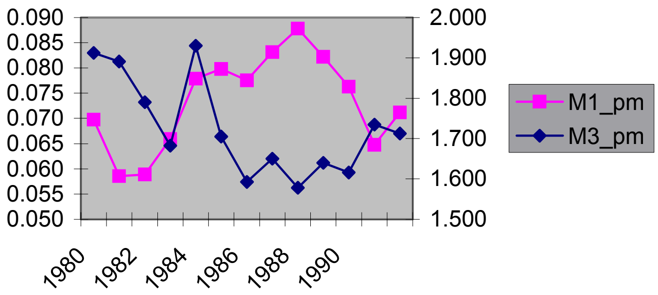






c) mean investment rate and M3, by asset





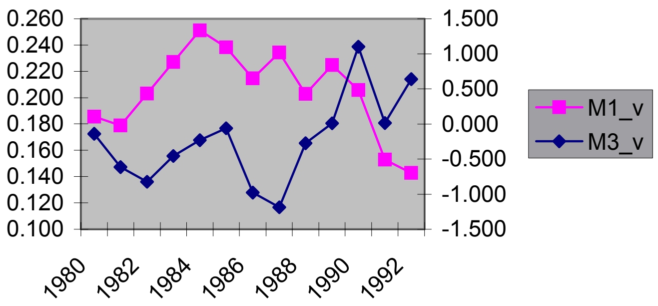


Figure 9: Nominal investment series, all assets. Manufacturing, m£

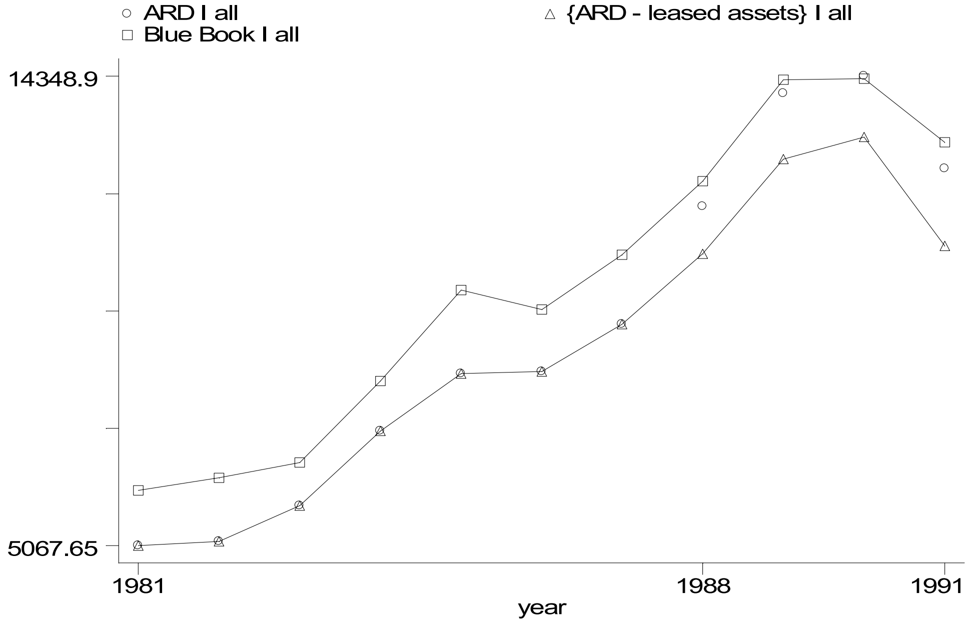


Figure 10 Comparison between Official Census nominal Investment series and investment series computed from ARD (including and excluding leasing).

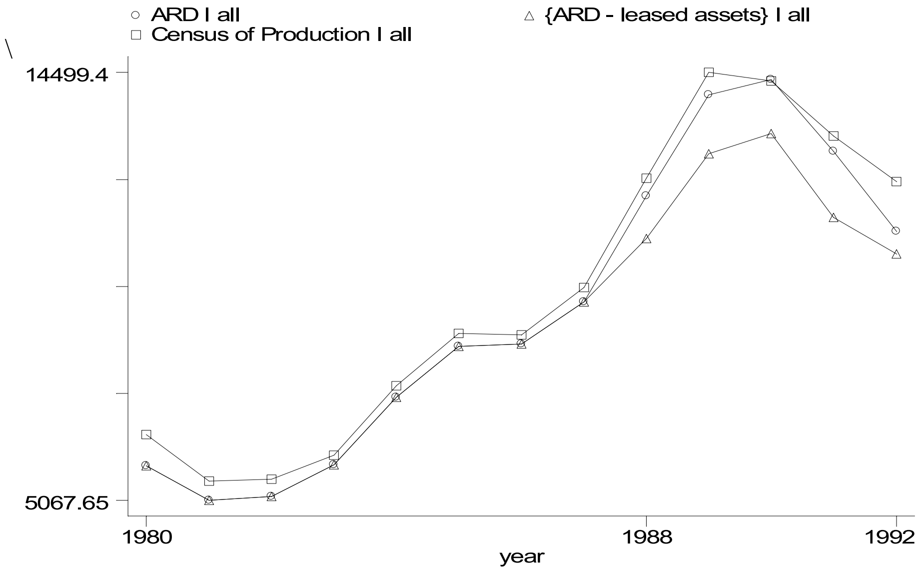
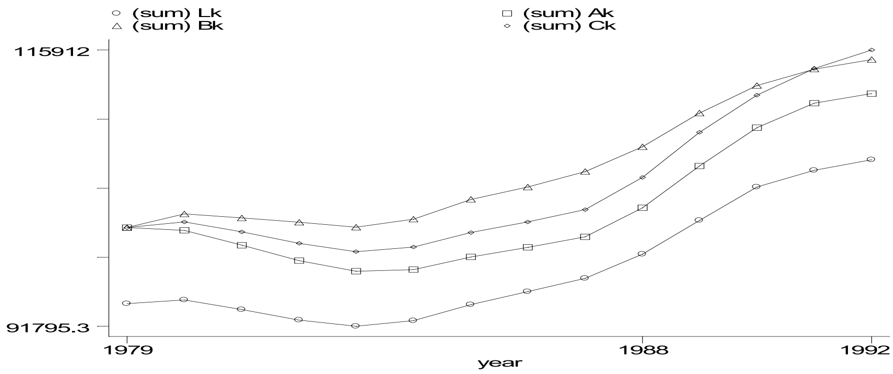


Figure 11: Real capital stock series



8. Tables

Table 1: Correlation coefficients between capital (K), employment (L) and output (Y), by size class

size class	20-50	50-100	100-500	500-1000	>1000
O:					
Y-L	0.2254	0.1187	0.3711	0.2169	0.7865
Y-K	0.3391	0.2739	0.4090	0.4623	0.7863
L-K	0.2239	0.1461	0.2935	0.1462	0.7006
A:					
Y-L	0.3438	0.2865	0.5373	0.2887	0.8123
Y-K	0.4408	0.4732	0.6402	0.5699	0.8786
L-K	0.3501	0.2914	0.5004	0.2492	0.7385
B:					
Y-L	0.4706	0.3870	0.6553	0.3719	0.8744
Y-K	0.4887	0.4921	0.7171	0.5662	0.9210
L-K	0.5041	0.4233	0.6359	0.3558	0.7947

O: including all observations
A: excluding first and last 1% of the distributions of K/Y, K/L and Y/L by year and size class
B: excluding first and last 5% of the distributions of K/Y, K/L and Y/L by year and size class
* all 0.0001 significant

Table 2: Distribution of investment rates

	bl				pm				v				all		
Investment Rates	# obs.	perc.	share	share in T	# obs.	perc.	Share	share in T	# obs.	perc.	share	share in T	# obs.	perc.	share
ir<0	8790	5.4%	-19.7%	-3.4%	6319	3.9%	-1.3%	-1.0%	22085	13.7%	-10.5%	-0.5%	10809	6.7%	-3.1%
ir=0	93463	57.8%	0.0%	0.0%	3651	2.3%	0.0%	0.0%	41626	25.7%	0.0%	0.0%	1965	1.2%	0.0%
0<ir≤δ	28870	17.9%	10.0%	1.7%	90190	55.8%	16.9%	13.1%	36686	22.7%	23.1%	1.2%	103001	63.7%	26.8%
δ<ir≤0.2+δ	21709	13.4%	48.6%	8.5%	50194	31.0%	57.9%	44.9%	20723	12.8%	28.9%	1.5%	37137	23.0%	51.6%
ir>0.2+δ	8863	5.5%	61.0%	10.6%	11341	7.0%	26.5%	20.5%	40575	25.1%	58.5%	3.0%	8783	5.4%	24.6%
Total	161695	100%	100%	17.4%	161695	100%	100%	77.4%	161695	100%	100%	5.2%	161695	100%	100%
δ<ir≤0.15+δ	19803	12.3%	40.2%	7.0%	45538	28.2%	49.2%	38.1%	16403	10.1%	22.6%	1.2%	33985	21.0%	45.8%
ir>0.15+δ	10769	6.7%	69.5%	12.1%	15997	9.9%	35.2%	27.3%	44895	27.8%	64.8%	3.4%	11935	7.5%	30.4%

Notes:
perc.: frequency of observations in each interval;
share: ratio of investment in each interval to total investment in the respective class of assets;
Share in T: percentage of total investment (sum of buildings and land, plant and machinery, vehicles) that is accounted for by the amount invested in each category for each interval of investment rate.
An example: take buildings and land; there are 28870 observations of investment rates between 0 and δ (# obs.) which correspond to 17.9% of total observations available. The amount of investment in buildings and land that corresponds to those 28870 investment rates is 10% of the total investment in buildings and land (all investment rates). The amount of investment in buildings and land that corresponds to those 28870 investment rates is 1.7% of the total investment in all assets (all investment rates, that is, the total amount of investment in the sample) – this is the share in T.

Table 3: Positive gross disposals vs. negative net investment rate.

	disposals>0			ir<0		
	#observations	# establishments	mean disposals/K	#observations	# establishments	mean |ir|
BL	14,271	7,874	.177	8,790	5,939	.221
PM	68,646	20,936	.021	6,319	5,246	.062
V	104,092	27,481	.417	22,085	13,943	.471
T	123,232	30,007	.022	10,809	8,454	.076

Table 4: Zero gross acquisitions vs. zero net investment rate

	Acquisitions=0		ir=0	
	#observations	# establishments	#observations	# establishments
BL	97,999	30,677	93,463	30,130
PM	4,152	2,868	3,651	2,534
V	53,613	22,686	41,626	18,927
T	2,546	1,817	1,965	1,357

Table 5: Contemporaneous occurrence of acquisitions and disposals at the plant level

size	20-50			50-100			>100		
disposals →	no	yes	Total	no	yes	Total	no	yes	Total
acquisitions ↓									
Building Land									
no	72.0	1.9	73.9	61.7	2.8	64.5	44.0	3.7	47.6
yes	22.6	*3.5*	26.1	30.4	*5.2*	35.5	41.6	*10.8*	52.4
Total	94.6	5.4	100.0	92.0	8.0	100.0	85.6	14.4	100.0
Plant Machinery									
no	2.0	0.2	2.2	2.0	0.5	2.5	1.9	0.5	2.4
yes	74.5	*23.3*	97.8	63.2	*34.3*	97.5	40.0	*57.7*	97.6
Total	76.5	23.5	100.0	65.2	34.8	100.0	41.9	58.1	100.0

Table 6: Distribution of zeros and spikes by size of establishment
a) percentage

Size	Zeros			Spikes		
	BL	PM	V	BL	PM	V
Total	57.8%	2.3%	25.7%	5.5%	7.0%	25.1%
20-50	76.0%	2.2%	34.9%	5.5%	6.7%	35.1%
50-100	65.7%	2.2%	26.5%	5.6%	6.6%	29.2%
100-1000	48.5%	2.5%	21.8%	5.5%	7.4%	19.4%
1000-10000	18.7%	0.4%	15.7%	5.3%	7.0%	11.8%
>10000	4.6%	2.0%	9.3%	2.0%	9.3%	10.6%

b) share of spikes and share in T

size	BL		PM		V	
	share	share in T	share	share in T	share	share in T
total	61.0%	10.6%	26.5%	20.5%	58.5%	3.0%
20-50	100.0%	24.5%	47.0%	28.5%	95.5%	14.2%
50-100	92.0%	20.7%	39.4%	26.1%	82.7%	9.3%
100-1000	73.0%	13.3%	32.0%	24.3%	58.2%	3.5%
1000-10000	46.6%	8.1%	19.4%	15.3%	42.2%	1.6%
>10000	6.7%	0.7%	19.9%	17.3%	51.4%	1.2%

Table 7: disposals by size

asset		20-50	50-100	100-500	500-1000	1000 +
BL	% zero disposal	94.56	92.01	88.14	80.87	69.35
	median disp/ks	0.043	0.031	0.022	0.014	0.007
PM	% zero disposal	76.48	65.2	46.95	26.4	14.25
	median disp/ks	0.016	0.009	0.005	0.003	0.002
V	% zero disposal	48.26	37.64	30.69	24.85	20.94
	median disp/ks	0.304	0.222	0.154	0.110	0.095
ALL	% zero disposal	40.26	27.85	17.16	8.77	5.42
	median disp/ks	0.015	0.010	0.006	0.004	0.004

Table 8: Spikes and Lags

gap between spikes	Buildings and Land	Plant and Machinery	Vehicles
0 years	19.2%	25.5%	23.1%
1 year	5.2%	6.9%	12.7%
2 years	4.5%	4.5%	9.2%
3 years	2.7%	3.9%	5.7%
4 years	2.7%	1.9%	3.0%
5 years or more	5.2%	3.8%	4.1%
no more spikes till end	17.5%	14.4%	6.5%
no more spikes till out of sample	43.1%	39.1%	35.6%
No. observations	1866	2903	8203

Table 9: Establishments in the “re-enter test”, by size class

	Number of establishments				Share of 5-95 pct outliers of K/Y dist.			
year	20-50	50-100	>100	all	20-50	50-100	>100	all
1986	32	101	143	277	9.38	6.93	6.29	6.86
1987	60	152	193	408	10.00	7.89	7.25	7.84
1988	57	136	179	373	10.53	15.44	7.82	10.99
1989	68	117	161	346	7.35	10.26	5.59	7.51
1990	13	21	158	192	0.00	4.76	5.70	5.21
1991	22	54	124	200	13.64	11.11	3.23	6.50
1992	18	43	81	143	11.11	9.30	11.11	10.49
all	270	624	1039	1939	9.26	10.10	6.54	8.05

Table 10: Median distance between extrapolated and estimated initial capital

year	20-50	50-100	>100
1986	0.063	0.005	-0.045
1987	0.110	-0.087	-0.126
1988	-0.054	-0.063	-0.078
1989	-0.049	0.026	-0.092
1990	-0.121	-0.143	-0.089
1991	0.220	-0.011	-0.101
1992	-0.037	0.297	-0.169
Total	0.004	-0.023	-0.101

STATISTICAL APPENDIX

Table 11: Number of establishments in the dataset, by year and size class

year / size	20-50	50-100	100-500	500-1000	>1000	Total
1980	2,509	2,919	6,873	1,095	770	14,166
1981	2,783	3,227	6,205	1,041	695	13,951
1982	2,980	3,128	5,917	988	624	13,637
1983	3,033	3,161	5,880	938	582	13,594
1984	5,036	4,279	5,955	916	572	16,758
1985	3,134	2,856	5,856	940	577	13,363
1986	2,886	2,653	5,859	918	549	12,865
1987	2,841	2,644	5,839	900	527	12,751
1988	2,980	3,039	5,916	932	538	13,405
1989	5,443	4,780	6,162	899	548	17,832
1990	3,107	3,033	5,927	931	557	13,555
1991	3,227	3,041	5,724	858	527	13,377
1992	2,821	2,704	5,319	791	482	12,117
Total	42,780	41,464	77,432	12,147	7,548	181,371

Table 12: median capital stock at the establishment level, by size class

year	20-50	50-100	100-500	500-1000	>1000
1980	335,163	727,825	2,155,032	8,008,801	22,900,000
1981	356,643	818,526	2,356,625	8,645,423	23,600,000
1982	369,145	847,168	2,432,213	9,037,635	23,800,000
1983	371,944	861,782	2,480,624	9,129,951	24,000,000
1984	377,635	858,152	2,489,771	9,294,659	25,400,000
1985	397,906	867,361	2,576,702	9,422,500	25,500,000
1986	413,459	927,511	2,652,272	9,671,534	26,300,000
1987	409,881	918,983	2,687,032	9,654,910	26,100,000
1988	426,596	931,177	2,683,414	9,962,573	27,000,000
1989	426,357	950,176	2,670,310	10,400,000	27,800,000
1990	439,331	981,957	2,856,877	10,600,000	30,200,000
1991	452,761	1,051,475	3,017,782	11,400,000	33,500,000
1992	467,710	1,113,194	3,199,135	12,100,000	35,100,000
% change 80-92	39.55	52.95	48.45	51.08	53.28

Table 13: median capital stock at the establishment level, by output quartiles and size class

size 20-50

year	q1	q2	q3	q4
1980	237,317	309,033	362,633	481,614
1981	254,834	334,249	391,351	508,807
1982	265,097	340,737	397,972	512,895
1983	262,861	344,098	409,220	521,987
1984	282,095	351,699	416,575	529,420
1985	286,860	381,039	426,425	564,871
1986	300,533	391,785	469,436	576,778
1987	311,043	379,661	451,788	555,889
1988	321,524	395,931	466,590	609,303
1989	320,666	401,908	474,412	560,184
1990	317,086	398,383	489,021	602,638
1991	334,754	419,218	480,764	626,329
1992	334,183	423,084	511,041	692,586
% change 80-92	40.82	36.91	40.93	43.81

size 50-100

year	q1	q2	q3	q4
1980	577,661	666,617	745,612	1,064,529
1981	624,599	726,975	860,218	1,289,586
1982	640,151	765,342	889,590	1,251,959
1983	647,444	772,492	941,401	1,278,483
1984	650,529	784,273	923,990	1,276,213
1985	622,743	788,997	949,184	1,407,915
1986	670,880	836,480	1,011,483	1,339,923
1987	653,197	845,335	998,367	1,376,078
1988	711,837	856,865	982,690	1,320,485
1989	707,800	868,328	997,303	1,310,720
1990	751,301	892,403	1,043,945	1,403,863
1991	794,105	977,553	1,120,279	1,500,554
1992	805,229	1,029,224	1,231,052	1,666,865
% change 80-92	39.39	54.40	65.11	56.58

size >100

year	q1	q2	q3	q4
1980	1,339,304	2,016,851	3,718,032	10,900,000
1981	1,433,000	2,166,306	4,062,663	11,700,000
1982	1,480,220	2,226,636	4,166,537	11,800,000
1983	1,488,629	2,273,861	4,167,122	12,000,000
1984	1,470,501	2,253,601	4,202,746	12,000,000
1985	1,512,557	2,340,847	4,341,572	12,500,000
1986	1,530,984	2,381,070	4,402,932	12,400,000
1987	1,553,028	2,442,827	4,475,506	12,000,000
1988	1,568,969	2,439,924	4,455,165	12,300,000
1989	1,612,680	2,392,885	4,373,904	12,400,000
1990	1,649,534	2,591,511	4,690,031	13,800,000
1991	1,730,101	2,793,283	5,021,927	14,400,000
1992	1,818,268	2,999,879	5,281,128	15,200,000
% change 80-92	35.76	48.74	42.04	39.45

Table 14: Median plant and machinery capital stock at the establishment level, by year and size class

a) value

year / size	20-50	50-100	100-500	500-1000	>1000
1980	180,081	400,095	1,227,619	4,429,560	13,400,000
1981	191,849	453,187	1,335,753	4,851,225	13,700,000
1982	198,532	462,699	1,371,762	5,121,811	13,900,000
1983	201,805	473,826	1,408,585	5,311,308	14,400,000
1984	205,741	475,340	1,427,893	5,482,051	15,000,000
1985	213,361	486,997	1,485,473	5,571,957	15,100,000
1986	225,749	528,884	1,543,479	5,763,058	15,800,000
1987	228,079	520,680	1,574,459	5,959,939	16,500,000
1988	241,218	529,373	1,589,401	6,014,585	16,800,000
1989	242,591	553,388	1,603,518	6,421,211	17,300,000
1990	249,016	563,596	1,704,792	6,916,065	18,600,000
1991	262,103	607,509	1,805,182	7,214,061	20,500,000
1992	277,822	654,503	1,918,651	7,698,698	21,700,000
% change 80-92	54.28	63.59	56.29	73.80	61.94

b) share on total capital

year / size	20-50	50-100	100-500	500-1000	>1000
1980	0.555	0.557	0.565	0.567	0.574
1981	0.554	0.558	0.566	0.569	0.576
1982	0.556	0.557	0.568	0.572	0.578
1983	0.554	0.558	0.573	0.581	0.581
1984	0.558	0.567	0.575	0.588	0.590
1985	0.557	0.566	0.583	0.599	0.594
1986	0.567	0.574	0.588	0.606	0.606
1987	0.572	0.576	0.590	0.611	0.614
1988	0.581	0.579	0.599	0.618	0.617
1989	0.586	0.590	0.603	0.631	0.629
1990	0.586	0.588	0.609	0.629	0.635
1991	0.592	0.595	0.609	0.637	0.639
1992	0.601	0.600	0.612	0.635	0.640
% change 80-92	8.40	7.72	8.28	12.00	11.40

Table 15: Median buildings and land capital stock at the establishment level, by year and size class

a) value

year / size	20-50	50-100	100-500	500-1000	>1000
1980	141,139	307,887	870,457	3,334,865	9,187,055
1981	150,866	339,122	948,406	3,585,306	9,498,888
1982	155,737	350,898	975,024	3,675,872	9,829,443
1983	157,262	352,700	986,042	3,635,023	9,958,800
1984	156,593	344,554	976,034	3,612,984	10,100,000
1985	163,344	346,630	985,316	3,533,842	10,200,000
1986	165,948	365,646	997,880	3,572,052	10,300,000
1987	160,740	364,266	989,692	3,519,950	9,606,398
1988	165,177	367,566	977,115	3,496,781	9,747,592
1989	163,606	359,085	964,227	3,543,488	9,940,312
1990	168,010	375,836	1,016,541	3,580,704	10,600,000
1991	173,962	398,336	1,067,893	3,894,446	11,100,000
1992	179,609	425,827	1,130,367	4,083,666	10,900,000
% change 80-92	27.26	38.31	29.86	22.45	18.65

b) share on total capital

year / size	20-50	50-100	100-500	500-1000	>1000
1980	0.414	0.413	0.411	0.410	0.406
1981	0.416	0.413	0.408	0.408	0.406
1982	0.413	0.414	0.406	0.405	0.404
1983	0.413	0.411	0.403	0.397	0.403
1984	0.408	0.404	0.399	0.392	0.393
1985	0.406	0.404	0.392	0.387	0.388
1986	0.400	0.399	0.389	0.377	0.378
1987	0.396	0.396	0.386	0.368	0.366
1988	0.391	0.396	0.382	0.364	0.361
1989	0.387	0.387	0.377	0.352	0.361
1990	0.387	0.391	0.372	0.354	0.355
1991	0.386	0.386	0.374	0.350	0.347
1992	0.387	0.388	0.374	0.351	0.342
% change 80-92	-6.53	-6.11	-9.02	-14.46	-15.85

Table 16: Median vehicles capital stock at the establishment level, by year and size class
a) value

year / size	20-50	50-100	100-500	500-1000	>1000
1980	8,988	17,575	45,456	161,661	403,276
1981	9,271	18,666	46,649	148,623	353,396
1982	9,388	18,980	45,067	134,297	329,194
1983	9,933	19,596	45,127	120,685	310,534
1984	10,133	19,963	44,862	117,903	300,039
1985	11,640	21,758	43,628	113,578	287,387
1986	11,451	20,936	41,786	108,050	264,177
1987	11,410	21,160	42,445	98,366	274,960
1988	9,860	19,346	39,206	98,799	249,646
1989	8,668	17,917	36,804	92,340	233,016
1990	8,564	17,988	35,167	96,456	226,044
1991	7,466	15,527	32,744	88,944	215,455
1992	5,476	12,552	28,690	73,436	176,528
% change 80-92	-39.08	-28.58	-36.89	-54.57	-56.23

b) share on total capital

year / size	20-50	50-100	100-500	500-1000	>1000
1980	0.029	0.026	0.021	0.020	0.017
1981	0.026	0.023	0.020	0.018	0.014
1982	0.025	0.023	0.018	0.015	0.012
1983	0.026	0.024	0.018	0.013	0.011
1984	0.026	0.024	0.018	0.012	0.010
1985	0.028	0.024	0.017	0.011	0.009
1986	0.027	0.023	0.016	0.010	0.008
1987	0.026	0.023	0.016	0.010	0.009
1988	0.023	0.021	0.015	0.009	0.009
1989	0.020	0.019	0.014	0.008	0.008
1990	0.019	0.018	0.012	0.008	0.006
1991	0.016	0.014	0.011	0.007	0.005
1992	0.013	0.012	0.010	0.006	0.005
% change 80-92	-56.28	-53.60	-55.21	-71.11	-71.58

Table 17: Median share of plant and machinery over total capital stock at the establishment level, by year, size class and output quartiles

size 20-50

year / output quintile	q1	q2	q3	q4
1980	0.549	0.556	0.554	0.561
1981	0.550	0.554	0.555	0.562
1982	0.547	0.564	0.554	0.553
1983	0.548	0.556	0.556	0.554
1984	0.554	0.561	0.563	0.555
1985	0.554	0.552	0.566	0.559
1986	0.556	0.571	0.571	0.568
1987	0.564	0.574	0.573	0.579
1988	0.569	0.580	0.585	0.590
1989	0.572	0.589	0.592	0.589
1990	0.577	0.588	0.590	0.592
1991	0.588	0.593	0.592	0.595
1992	0.588	0.601	0.607	0.614
% change 80-92	7.14	8.16	9.56	9.36

size 50-100

year / output quintile	q1	q2	q3	q4
1980	0.556	0.561	0.554	0.556
1981	0.552	0.559	0.558	0.562
1982	0.550	0.560	0.557	0.560
1983	0.544	0.561	0.559	0.565
1984	0.553	0.572	0.568	0.571
1985	0.553	0.567	0.570	0.573
1986	0.554	0.574	0.583	0.576
1987	0.564	0.578	0.581	0.584
1988	0.570	0.578	0.581	0.591
1989	0.576	0.589	0.596	0.600
1990	0.571	0.579	0.598	0.607
1991	0.580	0.594	0.606	0.601
1992	0.580	0.597	0.612	0.616
% change 80-92	4.31	6.26	10.53	10.77

size >100

year / output quintile	q1	q2	q3	q4
1980	0.558	0.566	0.568	0.571
1981	0.558	0.567	0.571	0.576
1982	0.556	0.563	0.575	0.582
1983	0.557	0.574	0.576	0.589
1984	0.554	0.571	0.584	0.600
1985	0.557	0.583	0.591	0.604
1986	0.564	0.590	0.599	0.612
1987	0.571	0.589	0.606	0.619
1988	0.576	0.598	0.611	0.627
1989	0.585	0.600	0.619	0.634
1990	0.586	0.610	0.624	0.639
1991	0.588	0.614	0.629	0.640
1992	0.589	0.615	0.629	0.644
% change 80-92	5.65	8.51	10.81	12.72

Building the capital stock

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1 Introduction

The ARD[1] does not contain information on capital stocks. If we still wish to use it for applications such as calculating TFP we have to calculate capital stocks with a perpetual inventory method (PIM). There various ways to do this. This document describes the method we used and discusses various alternatives.

2 Building the capital stock

2.1 Perpetual inventory

We use the following perpetual inventory formula: $k_t = k_{t-1}(1-d)+i_t$

where k represents the capital stock , d the geometric depreciation rate and i is investment.

2.2 Investment series

The basic ingredient to the perpetual inventory method is the investment information provided in the ARD. The ARD distinguishes between 3 types of investment:

1. Plant and Machinery
2. Buildings
3. Vehicles

We do a perpetual inventory calculation for each one of these asset types. Our measure of total capital stock is then obtained by summing across the 3 asset types.

Table 1 shows in detail which variables we used to calculate investment series. We use what in ARD terminology is net capital expenditure. This is capital expenditure minus proceeds from disposal of capital; i.e. this is not net of depreciation which the ARD does not have any information about.

[1] For a detailed description of the ARD see Barnes and Martin [1]

Table 1: Variables used as investment information

Variable	1980	1981-1992	1993-1995	1996-1997	1998	1999-2000
total net capital expenditure	(q154-q155)/1000	(q154-q155)/1000	q154-q155	q817-q818	q523	wq522+wq521
net capital expenditure for plant and machinery	(q517-q518)/1000	(q517-q518)/1000	q517-q518	q853-q854	q527-q530	wq527-wq530
net capital buildings	(q501+q502-q503)/1000	(q501+q502-q503)/1000	q501+q502-q503	q849+q848-q850	q524+q525-q528	wq524+wq525-wq528
net capital expenditure for vehicles	(q513+q515-q514-q516)/1000	(q504-q505)/1000	q504-q505	q851-q852	q526-q529	wq526-wq529

Notes: The cell entries refer to the ARD question numbers; i.e. total net capital expenditure in 1980 is obtained by subtracting question 155 from question 154 and dividing by 1000 to account for the unit change after 1992.

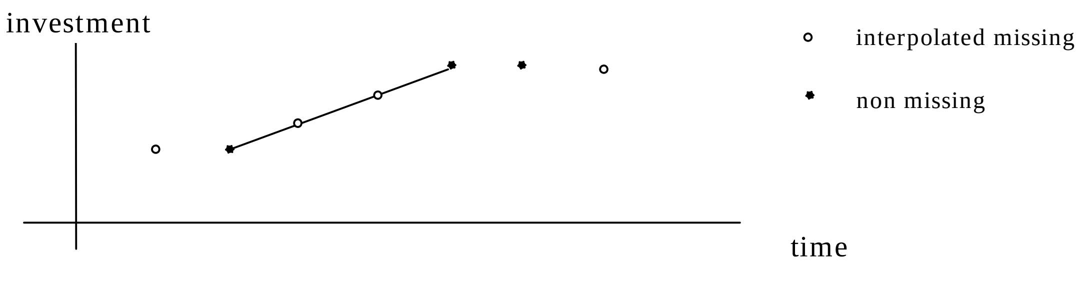
2.2.1 Gaps

The ARD surveys smaller units only on a random basis. If we want to calculate capital stocks for these we cannot just include only those years in the perpetual inventory method where the units were sampled. Because they are investing in the other years as well we would vastly underestimate their true capital stock. To avoid this we apply 3 types of interpolations:

1. We linearly interpolate the investment series in years where we have an observation both before and after the missing period,
2. set missing values at the birth of a unit equal to the first observed investment value and
3. set missing values at the death of a unit equal to the last observed.

Figure 1 shows this graphically.

Figure 1: Inter- and extrapolation of investment series

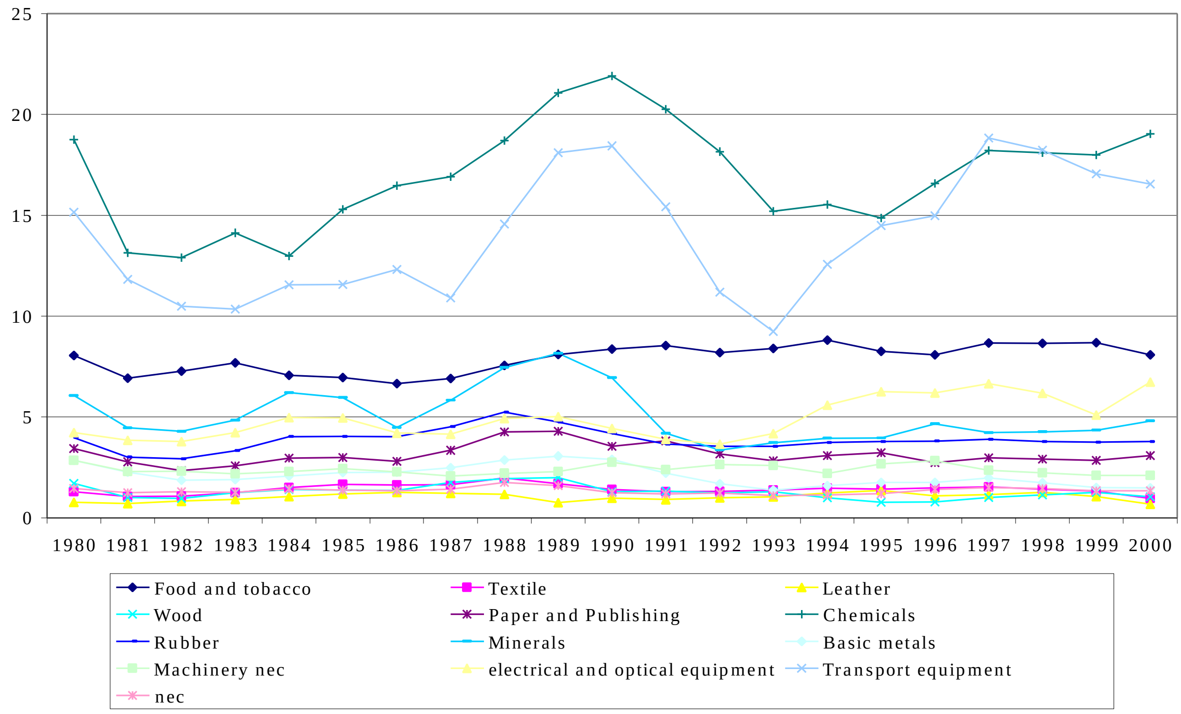


2.2.2 Other Problems

In 1994 and 1998 investment information by 3 asset types was missing from our data. For these years we interpolated the numbers as described in the last section.

Attanasio et al. [2] report about a structural break in investment reporting in 1988. According to them there has been a change in the treatment of leased assets in this year. Figure 2 shows average investment levels by sector as found in our dataset. As there is no apparent structural break in 1988 we assumed that the problem described by Attanasio et al. is not present in our release of the data.

Figure 2: Average investment by sector
(Aggregated from figures by 3 asset types)



2.3 Initial values

There are two problems with initial values. First, as our panel ranges from 1980 to 2000 units which are born before 1980 suffer from left censoring. Because they have accumulated capital before 1980 we would grossly underestimate their capital stock if we only considered their post 1980 investment.

Secondly, even units which are born after 1980 will have undertaken some initial investment before they first report to the ONS[2]. To take account of both issues we allocate initial capital stock values derived from sectoral aggregates at the 2 digit level. The sectoral aggregates are based on historical investment series by various asset types stretching back to 1948 provided by the ONS. To derive these stocks a perpetual inventory method with geometric depreciation has been applied on the sectoral level. We use real and nominal values of the same series to

get implied deflators.

Figure 3 Allocating initial capital stock

Aggregated capital stock	non Selected	
	Selected	Estab. 1
		Estab. 2
		Estab. 3
		….
		Estab N

To get initial values we have to estimate how much of the aggregate capital stock in a given year is due to the individual establishments in our selected[3] sample (see Figure 3). To do this we proceed in two steps. First, we have to estimate the share of aggregate sectoral capital stock which corresponds to the selected units in our sample. We estimate this as the investment share of a sectors selected units: $g_{Sector} = \frac{I_{Selected,SectorI}}{I_{SectorI}}$.

Secondly, we distribute this selected units capital stock among the selected new born or left censored units on a pro rata basis according to the average material usage over the lifetime of a unit. Hence establishment i gets a share g_i of the capital stock where

$$g_i = \frac{\frac{1}{T_i}\sum_{t=1}^{T_i} M_{t,i}}{\sum_{i \in Sector} \frac{1}{T_i}\sum_{t=1}^{T_i} M_{t,i}}$$

[2] Thiswas an adviced we received from ONS.

[3] to the selected units we count in this context also the who were not selected in a year but whose investment series have been interpolated as described in 2.2.1.

2.4 Investment deflators

We use investment deflators with base year 1995. For years pre 1995 these are implicitly derived from nominal and real sectoral ONS historical investment series (see also previous section). From 1995 on we use the publicly available MM17 series.

2.5 Depreciation rates

Depreciation rates have to be assumed we make the following assumption:

Asset Type	**Deprecation rate**
Plant and machinery	0.06
Building	0.02
Vehicles	0.2
Composite asset invesstment	average of above=0.11

These numbers are equal to the average depreciation rates used in PIM calculations for sectoral aggregates by the ONS.

3 Looking at the capital stock

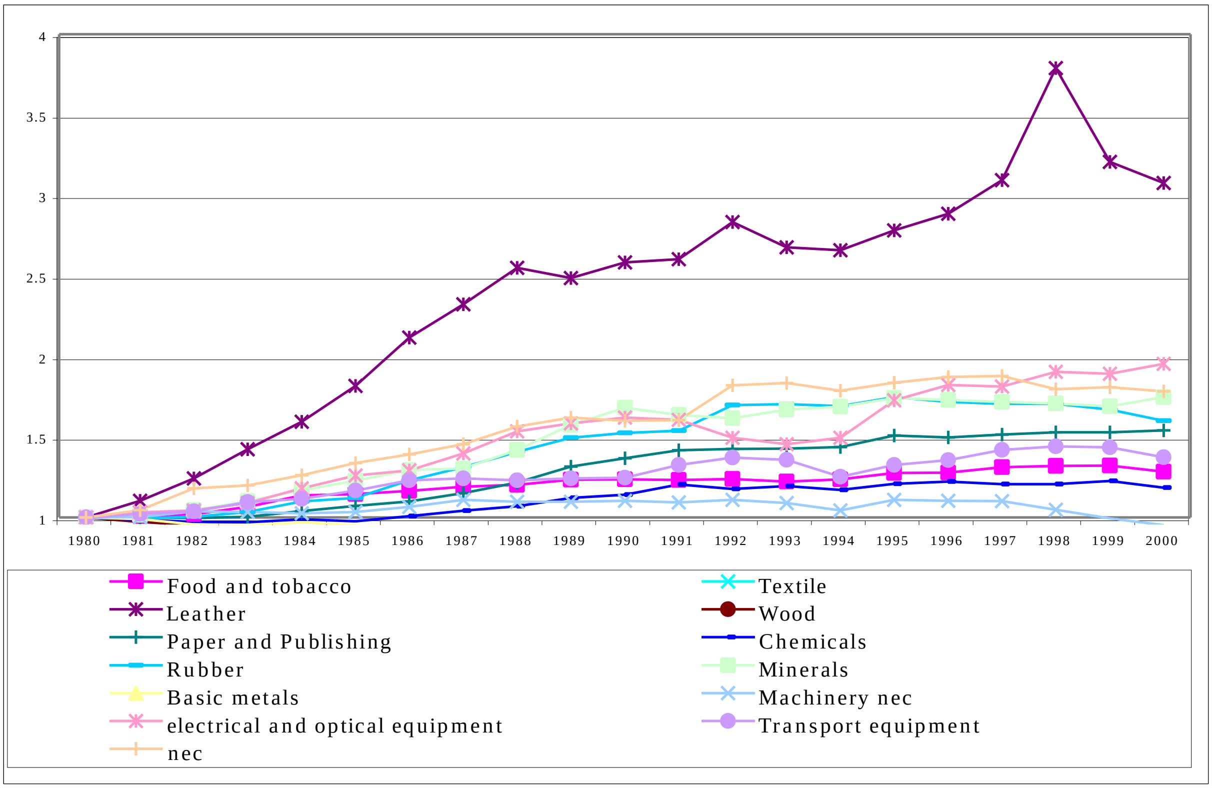
To asses the quality of our capital stock measure we use three criteria:

1. Aggregate our establishment level series and compare to the ONS aggregate series
2. Calculate TFP and annual transition matrices of the TFP distribution
3. Regress labour productivity on capital and examine the resulting coefficient.

3.1 Aggregates

For most purposes we will only need a composite capital stock measure. When building the capital stock however, we run perpetual inventory calculations on 3 asset types. Plant and machinery, buildings and vehicles. The composite capital stock is found in turn by aggregating these series. Figure 4 shows an index constructed from the so found sectoral aggregates of the capital stock measure.

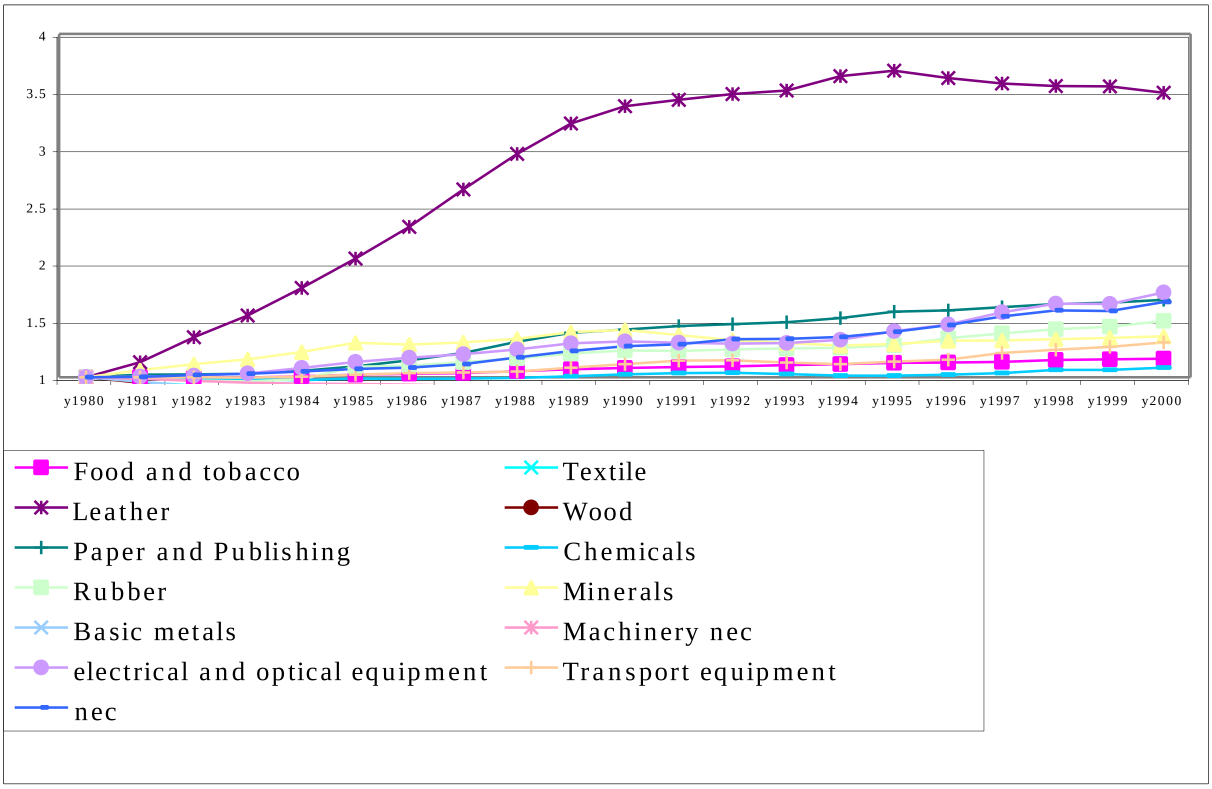
Figure 4: Index of real capital stock series by sector
(Permament inventory calculations based on microdata from ARD)



To get an idea if our capital stock makes sense we compare it with the sectoral aggregate capital stock series based on the ONS historical investment series. An index of this capital stock measure is displayed in Figure 5. The basic qualitative features of the two sets of series seem to be in line with each other. The measure based on ARD microdata seems more volatile which is natural given that there we also capture entry and exit which affect the aggregate stock in a very lumpy way.

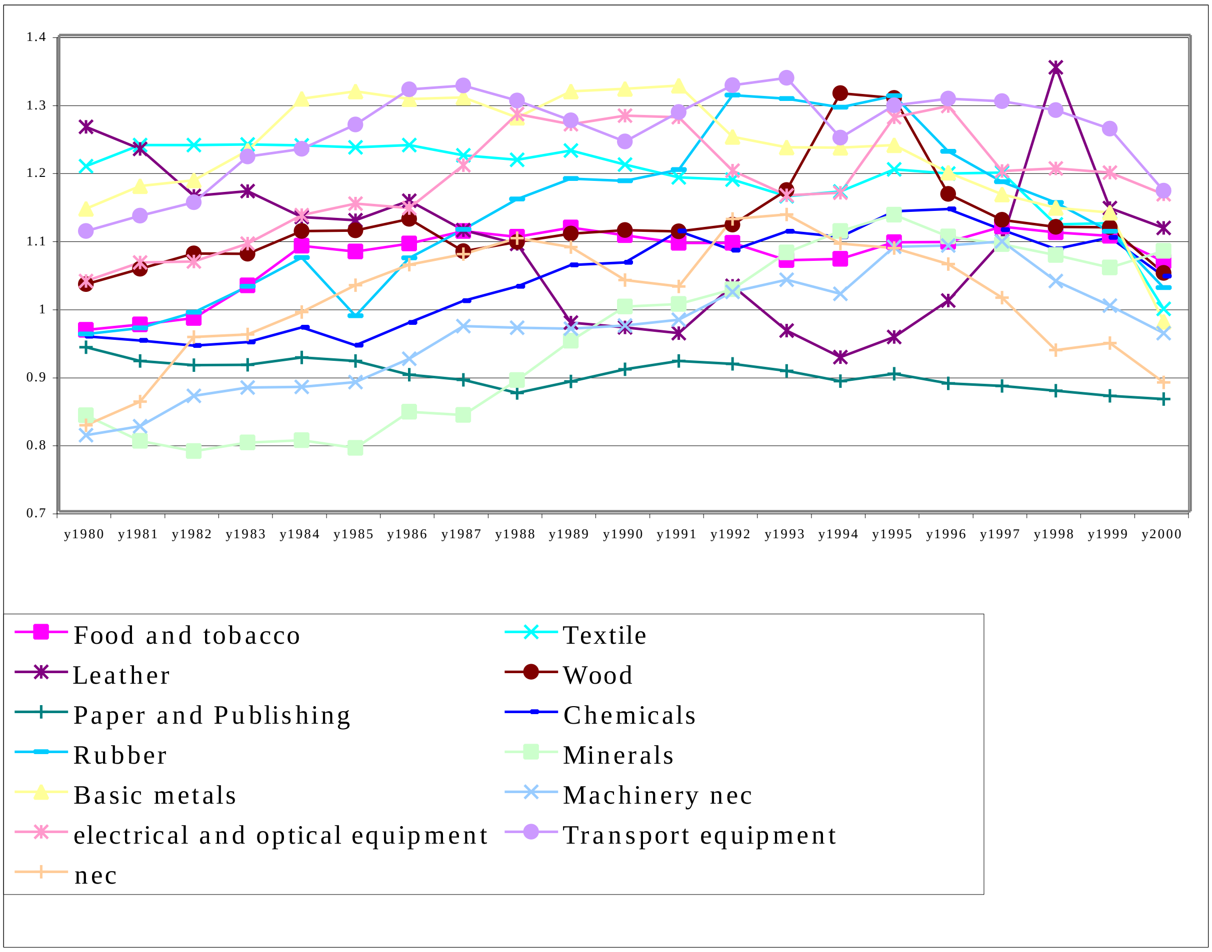
Figure 5: Index of real capital stock series by sector

(Permament inventory calculations based ONS historical aggregate investment series)



To further compare the two measures Figure 6 shows the ratios between our two sets of capital stocks. Note that we did not apply any weighting to the ARD series so it only represents the capital stock of selected units. It is therefore not surprising that the ratio is usually smaller than one.

Figure 6: ARD capital stock over Aggregate capital stock



3.2 *TFP and transition matrices*

Table 2 shows annual transitions matrices of the TFP[4] distribution. TFP is a measure of firm performance which can be related to the exit probability of establishments reported in the last column of this table. The exit probability is another measure of firm performance which

[4] Following Caves, Christensen and Diewert (1981) we calculate the TFP of plant *i* relative to the industry TFP as

$$\ln TFP_{it} = \ln Y_{it} - \ln Y_{It} - \bar{a}_K\left(\ln K_{it} - \ln K_{It}\right) - \bar{a}_L\left(\ln L_{it} - \ln L_{It}\right) - \bar{a}_M\left(\ln M_{it} - \ln M_{It}\right)$$

where *I* denotes industry and the factor shares are the average of the of the plant and median industry factor share; e.g. $\bar{a}_L = \dfrac{a_{L,i} + a_{L,I_{sic4dig}}}{2}$. Capital shares were calculated as the residual of labour and material share. If labour and material costs exceeded gross output we assigned the industry median as firm level factor shares.

is independent of our TFP calculation. If our capital stock measure and in turn our TFP measure make sense and the competitive selection process is working reasonably well then we should see that establishments with low TFP should have higher exit probability. Inspection of Table 2 shows that this is indeed the case: with 11% the bottom quintile has the highest exit probability, although exit probability is very high in all quintiles.

Table 2: Annual TFP Transitions

	20	**40**	**60**	**80**	**100**	**exit**
20	0.52	0.22	0.09	0.04	0.02	0.11
40	0.22	0.35	0.22	0.10	0.03	0.08
60	0.09	0.22	0.32	0.22	0.07	0.07
80	0.05	0.10	0.23	0.36	0.19	0.08
100	0.03	0.04	0.09	0.22	0.52	0.09
entry	0.24	0.19	0.18	0.18	0.21	

Source: Authors' calculations based on ARD

Notes: Row 1 column 2 shows for example the probability that an establishment whose TFP in t falls into the bottom quintile moves on to the 2nd quintile in t+1.

The entry row shows the fraction of entrants that have entered to the various quintiles.

3.3 *Productivity Regressions*

Regressing our capital stock on gross output over employement for a set of manufacturing establishment level data from 1980 to 2000 yields the following result:

$$\ln\left({}^{go}\!\!\diagup_{emp}\right)_{i,t} = S(i,t) + \underset{(0.001)}{0.083}\ln\left({}^{\mathrm{k}}\!\!\diagup_{\mathrm{emp}}\right)_{i,t} + \underset{(0.001)}{0.622}\ln\left({}^{mat}\!\!\diagup_{emp}\right)_{i,t} \underset{(0.001)}{-0.003}\ln\left(emp\right)_{i,t} + e_{i,t}$$

where $S(i,t)$ represents a set of sector dummies and the point estimate standard errors are reported in parenthesis.

The lower the quality of our capital stock the smaller we expect the coefficient on the capital stock to be. The value of 0.083 for the capital coefficient is still in the range of what we would expect. From aggregate data we would expect a value of 0.15 which is the average capital share in gross output.

4 Alternative ways to calculate the capital stock

4.1 *Treatment of initial values*

We experimented with allocating initial capital stocks only to units which were born before 1980. We thought that such an underestimation of capital stock might be preferable to the crude allocation of initial values to post 1980 entrants from some aggregate value. It turned out however that it is crucial to get plausible results. Table 3 shows annual transition matrices of TFP calculated with a capital stock series as in Table 2 before but this time without allocating initial values post 1980. The transition matrix for the series without initial values (Table 3) has implausible concentration of entries with very high productivity and an exit rate from the top quintile as high as for the bottom. This is most likely a consequence of the capital stock underestimation post 1980: Newly entering establishments which are more likely to exit have an overestimated TFP.

Table 3: Annual TFP Transitions without initial values post 1980

	20	40	60	80	100	exit
20	0.57	0.21	0.08	0.03	0.01	0.10
40	0.23	0.38	0.21	0.08	0.02	0.07
60	0.09	0.24	0.35	0.20	0.05	0.07
80	0.04	0.10	0.24	0.39	0.15	0.08
100	0.02	0.03	0.08	0.24	0.53	0.10
entry	0.13	0.11	0.13	0.18	0.44	

Further we experimented with allocating initial values on the basis of an establishments average share in aggregate investment (instead of materials). Investment could potentially be a better estimator for the level of capital stock. It turned out that taking material shares is again crucial to get plausible results when calculating TFP, however. Table 4 shows a TFP transition matrix using a capital stock measure calculated on the basis of investment shares. Again there is a problem with entry and exit rates.

Table 4: Transition Probabilities for ln_TFP_go_mean_sep02_yearly

	20	40	60	80	100	exit
20	0.62	0.21	0.06	0.02	0.01	0.08
40	0.21	0.41	0.23	0.07	0.02	0.07
60	0.06	0.24	0.38	0.21	0.04	0.07
80	0.02	0.07	0.24	0.43	0.15	0.08
100	0.01	0.02	0.06	0.22	0.59	0.11
entry	0.19	0.12	0.14	0.18	0.37	

Source: Authors' calculations based on ARD

Notes: Row 1 column 2 shows for example the probability that a firm whose TFP in t

Things get even worse when using both: investment shares and no initial values post 1980 (Table 5).

Table 5: Annual transition matrix for TFP

(TFP relative to 4 digit sector mean, based on gross output)

	20	40	60	80	100	exit
20	0.65	0.20	0.05	0.02	0.01	0.08
40	0.21	0.42	0.21	0.07	0.02	0.07
60	0.06	0.24	0.38	0.20	0.04	0.07
80	0.02	0.08	0.25	0.41	0.15	0.08
100	0.01	0.02	0.07	0.24	0.55	0.11
entry	0.13	0.11	0.13	0.19	0.45	

Source: Authors' calculations based on ARD

Notes: Row 1 column 2 shows for example the probability that a firm whose TFP in t falls into the bottom quintile moves on to the 2nd quintile in t+1.

The entry row shows the fraction of entrants that have entered to the various quintiles.

Table 6 reproduces the productivity regression of Section 3.3 in column 1. Column 2 shows the same regression using the capital stock measures with initial values allocated on the basis of investment shares. The capital coefficient is with 0.031 much further away from our prior of 0.15. This confirms once more that the measure used by us is superior to alternative specifications.

Table 6: Productivity regression

(Using various capital stock measures)

	(1) mat share k	(2) inv share k
lnkl	0.084	0.031
	(0.001)***	(0.000)***
lnml	0.622	0.698
	(0.001)***	(0.001)***
lnl	-0.003	0.011
	(0.001)***	(0.001)***
Observations	215315	190512
R-squared	0.90	0.92

Standard errors in parentheses

* significant at 10%; ** significant at 5%; *** significant at 1%

4.2 Accounting for plant closures

Richard Harris (Harris and Drinkwater 2000) in particular has pointed out that for multi-plant establishments plant closures could lead to an overestimation of the capital stock with the perpetual inventory method. With the perpetual inventory method we assume a constant depreciation rate which does not account for the discrete drop in an establishments capital stock in connection with a plant closure. We have not accounted for plant closures in our calculations for three reasons:

1. To be able to account for plant closures we need to have an estimate of the size of the capital stock at individual plants. We eschewed the strong assumptions needed to produce such an estimate. The problem is that for multi-plant establishments the only data available at the plant level is employment. Thus to allocate investment and capital we have to assume a constant investment labour ratio across the local units of an establishment. Moreover, for many[5] local units the employment figure itself is simply an interpolation by the ONS.
2. The additional work required to account for plant closures is considerable
3. It is not clear if accounting for plant closures would make a great difference to our micro level results.

We have two pieces of evidence regarding this last point: First, as a matter of fact about 30% of all selected plants belong to single plant establishments (about 50% of all plants are in establishments with less than 4 plants). About 50% of the employment in selected firms is in single plant establishments. Second, Harris (2000) uses an ARD local unit dataset which takes account of plant closures when calculating the capital stock. Harris finds that his results on the productivity difference between foreign owned and domestic firms differ from a similar study by Griffith (1999) who worked on the establishment level. Comparing his Tables 2 and A2 suggests however that the differences were mainly driven by using weighted regressions rather than local unit data.

[5] Employment information on the non-selected file (Perry, 1994) differs depending on whether collected before or after 1994. Before 1994 for the 0-19 employment, it was interpolated using turnover data (usually derived from VAT data) and the turnover/employment ratios for the 20--49 employees band. The ONS did check employment for plants with imputed employment of over 11, but this was only around 20% of the non-selected sample and as for the imputed data due to time lags in the provision of tax data and processing of imputations, such information is typically refers to data from two years earlier (Perry, 1985). After 1994 it is from the IDBR. If there is PAYE information, then employment comes directly If there is only VAT turnover information, then employment is again interpolated.

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Capital stocks, capital services, and depreciation: an integrated framework

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Contents

Abstract

Neo-classical theory provides an integrated framework by means of which we can measure capital stocks, capital services and depreciation. In this paper the theory is set out and reviewed. The paper finds that the theory is quite robust and can deal with assets like computers that are subject to rapid obsolescence. Using the framework, estimates are presented of aggregate wealth, aggregate capital services and aggregate depreciation for the United Kingdom between 1979 Q1 and 2002 Q2, and the results are tested for sensitivity to the assumptions. We find that the principal source of uncertainty in estimating capital stocks and capital services relates to the treatment and measurement of investment in computers and software. Applying US methods for these assets to UK data has a substantial effect on the growth rate of capital services and on the ratio of depreciation to GDP.

Key words: capital stocks, capital services, depreciation

JEL classification: E22, O47

Summary

This paper presents an integrated framework to measure capital stocks, capital services, and depreciation. The framework is integrated in two senses: first, our approach to measuring each of these variables is intellectually consistent; second, we use a common set of data for all three variables. Much of the difficulty of deriving good measures of aggregate capital, whether stocks or services, derives from two basic empirical facts. First, the relative prices of different types of asset are changing. Second, the pattern of investment is shifting towards assets with shorter economic lives. So we cannot treat capital as if it were composed of a single homogeneous good. To some extent, these two facts are aspects of the same important economic change: the shift in the pattern of investment towards information, communications and technology (ICT) assets. The relative prices of these assets are falling rapidly and their economic lives are much shorter than those of most other types of plant and machinery.

Theory

The wealth concept of capital, while appropriate for some purposes, is not the right one for a production function or for a measure of capacity utilisation. For the latter purposes, we need a measure of aggregate capital *services*. A second concept of aggregate capital, which will be called here the volume index of capital services (VICS), answers this need.

In principle, the VICS measures the flow of capital services derived from all the capital assets, of all types and all ages, that exist in a sector or in the whole economy. Methodologically, the main difference between the VICS and wealth-type measures of capital is the way in which different types and ages of assets are aggregated together. In the VICS, each item of capital is (in principle) weighted by its rental price. The rental price is the (usually notional) price that the user would have to pay to hire the asset for a period. By contrast, in wealth measures of the capital stock each item is weighted by the asset price.

An important practical implication of using a VICS rather than a wealth measure is that the VICS will give more weight to assets like computers and software for which the rental price is high in relation to the asset price.

We review the theory of, and empirical evidence on, depreciation. The assumption that depreciation is geometric greatly simplifies the theory and seems consistent with the (limited) facts. We also consider whether the geometric assumption is appropriate for assets like computers. Computers do not suffer much from physical wear and tear, but nevertheless have very short lives due to what is usually called ‘obsolescence’. We find that, in principle, our framework encompasses obsolescence. Nevertheless, we show that in practice depreciation rates may be somewhat overstated owing to failure to control fully for quality change.

Empirical measures of wealth and VICS

We adopt the geometric assumption in our empirical work for the United Kingdom. Because of the uncertainty about asset lives and the pattern of depreciation in the United Kingdom, we calculate wealth and VICS measures under a range of assumptions. We test the sensitivity of our

results in three main ways. First, we compare results using both US and UK assumptions about asset lives. Second, we compare results based on a comparatively coarse breakdown of assets into four types only, with results derived from a more detailed breakdown in which computers and software are distinguished separately. Third, we compare the effect of US versus UK price indices for computers and software. Our results are for the whole economy and all fixed assets excluding dwellings, for the period 1979 Q1-2002 Q2. Our main findings for wealth and VICS are as follows:

1. Using the conventional National Accounts breakdown of assets into buildings (excluding dwellings), plant and machinery, vehicles, and intangibles, we find that the *growth rates* of wealth and the VICS are insensitive to variations in depreciation rates (ie, asset lives). In these experiments the rates for each asset are assumed constant over time.
2. However, the *level* of wealth is quite sensitive to variations in depreciation rates.
3. Still sticking with the conventional asset breakdown, wealth and VICS grew at similar rates over the period as a whole. In the 1990s, the gap between the two measures widened a bit, with the growth rate of the VICS higher by about 0.1 percentage points per quarter.
4. The effect on the estimates of separating out computers and software is quite complex. First, much larger differences appear between the growth rates of VICS and wealth, of the order of 0.2-0.4 percentage points per quarter. Second, the growth rate of wealth tends to be slower, though that of the VICS is not necessarily faster. But when we apply the set of assumptions closest to US methods, the growth rate of the VICS is raised by 0.2 percentage points per quarter, relative to the VICS with computers and software included with other asset classes.

These results suggest that the treatment and measurement of investment in computers and software is an empirically important issue. It is common ground that the relative price of these assets has been falling, so it is in principle correct to separate them out explicitly – and it matters in practice. The conclusions about the growth rates of both VICS and wealth turn out also to be sensitive to the price index used for computers and to the way in which the level of software investment is measured.

The wealth and VICS estimates under a variety of assumptions can be downloaded from the Bank of England's website (www.bankofengland.co.uk/workingpapers/capdata.xls).

The aggregate depreciation rate and the ratio of aggregate depreciation to GDP

We also estimate aggregate depreciation (capital consumption) for the same range of assumptions. We study the sensitivity of the aggregate depreciation rate and of the ratio of depreciation to GDP to the assumptions, and compare our estimates with ones derived from official data. We find:

1. Using the conventional asset breakdown and our assumptions about depreciation rates at the asset level, there is no tendency for the aggregate depreciation rate to rise over the past two decades.

2. Separating out computers and software has less effect than one might have expected: even the use of US methods raises the aggregate rate by only about 1 percentage point, to 7% in 2000, and again there is no sign of an upward trend. The reason is that, even by 2000, the share of computers and software in wealth was only about 4% in the United Kingdom. By contrast and on a comparable basis, the aggregate depreciation rate in the United States has trended smoothly upwards since 1980, to reach nearly 9% in 2000. This illustrates the much greater scale of ICT investment in the United States.
3. Assumptions about asset lives have a large impact on the estimated ratio of depreciation to GDP. The official UK National Accounts measure has been drifting down fairly steadily since 1979. In 2001 it stood at 8%. Using shorter US asset lives and the conventional asset breakdown, the ratio was over 10% in the same year. Separating out ICT assets and using US methods, the ratio rises to nearly 13%, similar to the ratio in the United States. Interestingly, in neither country was there any upward trend in the ratio, except perhaps in the past couple of years. The reason is that, although the quantity of high-depreciation assets has been growing faster than GDP, this has been offset by their falling relative price.

1 Introduction

Capital is an important part of the economy. Together with labour, it is a key factor of production, contributing to the output the economy can produce; changes in it – investment – constitute an element of demand in the economy; and it constitutes wealth, from which its owners obtain income in the form of profit.

But capital can be defined in different ways: in the context of production theory, the correct concept is the flow of capital services, whereas in the context of wealth, the correct concept is the present value of the returns accruing from the capital over its remaining productive life. And capital is difficult to measure. An economy's capital is composed of different asset types and different vintages, and both the value of, and the services provided by, those assets change over time – eventually, to the point at which the asset has no further value or productive use. It is impossible in practice to measure those characteristics directly for each asset. So empirical measurement typically relies on measuring the rate at which new assets are acquired (gross investment) and the price of those new assets, and making a range of assumptions about how the quantity and value of older assets changes over time (loosely, depreciation).

In this paper we present an integrated framework to measure capital stocks, capital services, and depreciation, and apply it to the United Kingdom, illustrating the empirical differences which flow from the alternative concepts and different assumptions which can be made. The framework is integrated in two senses: first, our approach to measuring each of these variables is intellectually consistent; second, we use a common set of data for all three variables. Our approach is broadly neo-classical, in the tradition of Hall, Jorgenson, Griliches and Hulten. In the theoretical parts of this paper, we show that this framework is more robust than it is sometimes given credit for. Much of the difficulty of deriving good measures of aggregate capital, whether stocks or services, derives from two basic empirical facts. First, the relative prices of different types of asset are changing. Second, the pattern of investment is shifting towards assets with shorter economic lives. Because of these two facts, we cannot treat capital as if it were composed of a single homogeneous good. To some extent, though not entirely, these two facts are really aspects of the same important economic change: the shift in the pattern of investment towards information and communications technology (ICT) assets. The relative prices of these assets (at least on some measures) are falling rapidly and their economic lives are much shorter than those of most other types of plant and machinery.

Capital wealth and capital services

In current prices, the wealth represented by capital is just the sum of the values of the various asset stocks. Each stock is the cumulated sum of past investment, less the cumulated sum of depreciation (inclusive of retirement and scrapping), all revalued to current prices. In constant prices, the growth of wealth is a weighted average of the growth rates of the asset stocks, where the weights are the base-period shares of each asset in the value of wealth. Since the value of each asset is its price times its quantity, we refer to these kinds of weights as asset price weights.

Theory suggests that the wealth concept of capital, which we call for short the wealth stock or just wealth, is not the right one for a production function or for a measure of capacity utilisation.

For the latter purposes, we need a measure of aggregate capital *services*. A second concept of aggregate capital, which will be called here the volume index of capital services (VICS), answers this need.(1)

In principle, the VICS measures the flow of capital services derived from all capital assets, of all types and all ages, that exist in a sector or in the whole economy. Methodologically, the main difference between the VICS and wealth-type measures of capital is the way in which different types and ages of assets are aggregated together. In the VICS, each item of capital is (in principle) weighted by its rental price. The rental price is the (usually notional) price that the user would have to pay to hire the asset for a period. By contrast, in wealth measures of the capital stock each item is weighted by the asset price. The two types of price are of course related: the price of an asset should equal the discounted present value of its expected future rental prices.

An important practical implication of using a VICS rather than a wealth measure is that the VICS will give more weight to assets for which the rental price is high in relation to the asset price. The rental price to asset price ratio is high when depreciation is high, due to a short service life, or when the asset price is falling, so that holding the asset incurs a capital loss. If the stocks of such assets are growing more rapidly than those of other types, then the VICS will be growing more rapidly than the wealth stock. This is likely to be particularly the case at the moment, with the increasing importance of computers and similar high-tech assets that are characterised by rapid depreciation and falling prices.

Previous studies

The wealth measure of the capital stock is the more firmly established and is the standard measure produced by national statistical authorities, including the Office for National Statistics (ONS) in the United Kingdom. Statistical agencies commonly estimate two different measures of the aggregate capital stock, known generally as the gross stock and the net stock. Several different asset types may be distinguished, eg buildings, plant and machinery, vehicles, etc. Conceptually, the gross stock of any asset is simply the sum of the past history of gross investment in that asset in constant prices, less the sum of past retirements. The aggregate gross stock is just the sum of the gross stocks of the different assets. The net stock differs from the gross stock in that allowance is also made for depreciation, often at a straight-line rate over each asset's known or assumed service life.

In estimating stocks, statistical agencies nearly always employ what is called the perpetual inventory method (PIM). This starts with estimates of investment by asset and by industry or by sector. Capital stocks are then calculated by cumulating the flows of investment and subtracting estimated depreciation and retirements. Depreciation is not generally known directly, but is calculated by applying estimates of depreciation rates to the stocks. Depreciation rates may be based on asset lives (the straight-line method) or they may be deduced from econometric studies of new and second-hand asset prices (of which the best known are Hulten and Wykoff (1981a)

(1) The OECD capital stock manual (OECD 2001b) uses the term 'volume index of capital services', from which we have coined the acronym VICS. The VICS is often called the productive capital stock (by contrast with the wealth stock), but this term is highly misleading since it not a stock at all but a flow.

and (1981b)). Retirements are also not observed directly but can be calculated from estimates of the service lives of assets. Asset lives are usually derived from tax records and from surveys.[2]

Although the wealth concept is better known, the VICS concept is not new: it came to prominence in the seminal growth accounting study of Jorgenson and Griliches (1967) and was employed in subsequent studies by Jorgenson and his various collaborators, eg Jorgenson *et al* (1987) and Jorgenson and Stiroh (2000). The theory was set out in Jorgenson (1989); a related paper is Hall and Jorgenson (1967) on the cost of capital. Recently, the OECD has published a manual on capital measurement which contains a full discussion of the various concepts including the VICS, together with advice on how to measure it in practice (OECD (2001b)).

Versions of the VICS are already produced officially for the United States by the Bureau of Labor Statistics and for Australia by the Australian Bureau of Statistics. As far as the United Kingdom is concerned, unofficial versions of the VICS have previously been estimated by Oulton and O'Mahony (1994) for 128 industries within manufacturing (for three asset types: plant & machinery, buildings and vehicles) and by O'Mahony (1999) for 25 sectors covering the whole economy (for two asset types: plant & machinery and buildings). Oulton (2001a) contains annual estimates of the aggregate VICS incorporating explicit allowance for ICT assets. Earlier work at the Bank on the VICS is summarised in Oulton (2001b). Work is also currently under way at the ONS to produce a VICS on an experimental basis.

Plan of the paper

Sections 2 and 3 constitute the theoretical part of the paper. In Section 2 we start by reviewing the relevant part of capital theory. We discuss the relationship between asset prices and rental prices and show how this can be used to illuminate the twin issues of aggregating over vintages and aggregating over asset types. We also discuss the relationship between depreciation (how asset prices change with asset age) and what we call decay, which describes how the *services* of an asset change with age. Next, the equations of the two models used for estimating the VICS on quarterly and annual data are set out. These models make use of an important simplifying assumption, namely that depreciation is geometric. We compare the index number of the wealth measure with that of the VICS.

Section 3 is devoted to the related concepts of depreciation and replacement. Replacement is what must be spent to maintain the volume of capital *services* at the existing level, while depreciation is what must be spent to maintain the value of the capital *stock* at the existing level. We discuss the relationship between these two concepts and show that replacement and depreciation are equal when depreciation is geometric. We start by considering alternative measures of the aggregate depreciation rate. There are two broad classes of measure: nominal

[2] Three other methods of estimating capital stocks have been employed. First, it is possible to do a sample survey or even a census of capital stocks. Such a survey has recently been done for the United Kingdom but no results have as yet been published (West and Clifton-Fearnside (1999)). Second, fire insurance values have been employed (Smith (1986)). Third, stock market values have been used (Hall (2001)). None of these methods has gained general acceptance, so they will not be considered further here. Also, stock market values can only yield a wealth measure, not a VICS. In the academic literature depreciation rates have also been derived as a by product of estimating a production function (Prucha (1997)) and scrapping has been estimated from company accounts (Wadhwani and Wall (1986)).

and real. We show that the nominal measure is consistent with economic intuition, while the real measures may behave in counter intuitive ways. For example, when a chain index is used, the aggregate real rate may rise without limit. Next, we compare straight-line with geometric depreciation. Straight-line depreciation is not a very attractive assumption empirically, but the comparison is important because many statistical agencies (including the ONS) employ the straight-line assumption. We calculate the geometric rate, which is equivalent to straight-line depreciation in a steady state, for a range of values of the service life and the steady state growth rate.

Then we turn to the vexed issue of obsolescence. We discuss the appropriate measure of depreciation when assets are subject to obsolescence. We show that obsolescence makes little difference in theory, but that it does complicate the estimation of depreciation. However, an appropriately specified hedonic pricing approach can in principle deliver good estimates of the rate of depreciation.

The remainder of Section 3 reviews the evidence on the pattern of depreciation and on the length of asset lives, for the United Kingdom and the United States. We discuss the depreciation rates used by the U.S. Bureau of Economic Analysis (BEA). We find that, as measures of *economic* depreciation in the neo-classical sense, their rates may be too high. Quantitatively, the largest divergence relates to personal computers (PCs). The BEA assumes a rate of about 40% per annum, while the study on which they rely suggests a rate of about 30% per annum as a measure of economic depreciation.

Section 4 sets out our estimates for the United Kingdom. We describe our sources and methods before going on to present our estimates for the wealth stock, the VICS, and aggregate depreciation, for a range of assumptions about depreciation and service lives, and for different degrees of disaggregation by asset type. We consider the sensitivity of our estimates to our assumptions. Finally, Section 5 concludes.

2 Theory of capital measurement

This section shows how in principle wealth and VICS can be measured from data on investment flows, asset prices and depreciation rates. There are two major theoretical issues to be settled: first, how to aggregate over vintages of a given type of asset, and second, how to aggregate over different asset types. In this section, we establish first of all the relationship between rental prices and asset prices. Then we apply this relationship to resolving these two issues.[3]

Asset prices and rental prices

Consider a leasing company that buys a new machine at the end of period t-1 and rents it out during period t. It pays a price $p^A_{t-1,0}$, where the superscript 'A' indicates this is an asset price.

[3] Our treatment draws heavily on Jorgenson (1989); see also Diewert (1980). Papers that focus on depreciation include Hulten and Wykoff (1996) and Jorgenson (1996). An exhaustive discussion of the concept of the VICS, together with a summary of research in this area, empirical findings and the practices of national statistical agencies, is in the OECD manual on measuring capital (OECD (20001b)); a shorter treatment is in the OECD productivity manual (OECD (2001a)).

The first subscript indicates the time at which the asset is acquired, the second the asset's age (zero in this case, since it is new). By definition, the value of the leasing company's investment one period later, at the end of period *t*, is $(1+r_t)\cdot p^A_{t-1,0}$ where r_t is the actual nominal rate of return during period *t* (this may differ from the equilibrium rate of return). What does the return actually consist of? During period *t* the leasing company rents out the asset and at the end of *t* it is paid a rental which we write as $p^K_{t,0}$. Here the first subscript denotes the period in which the rental is received and the second the asset's age. The superscript '*K*' indicates that this is the *rental* price for capital services (*K*), as opposed to the *asset* price (denoted by a superscript '*A*'). At the end of period *t*, the leasing company has an asset which is now one year old and which can (if desired) be sold for a price $p^A_{t,1}$. So the value of the leasing company's investment is (ignoring tax for the moment):

$$(1+r_t)\cdot p^A_{t-1,0} = p^K_{t,0} + p^A_{t,1} \qquad \textbf{(1)}$$

Iterating this equation forward, we obtain:

$$p^A_{t-1,0} = \sum_{z=0}^{n}\left[p^K_{t+z,z} \Big/ \prod_{\tau=0}^{z}(1+r_{t+\tau}) \right] \qquad \textbf{(2)}$$

assuming the asset is valueless at the end of its assumed life of *n* periods. That is, the asset price equals the present value of the future stream of rental prices.

From the point of view of the firm to which the leasing company rents the asset, the rental price is what it must pay for the use of the machine's services for one period. A profit-maximising firm will hire machines up to the point where the rental price equals the marginal revenue product of the machine. Under perfect competition, the rental price will equal the value of the marginal product: the output price multiplied by the machine's marginal *physical* product. So under these assumptions the rental price measures the contribution of the machine to producing output.

Though financial leasing is a common arrangement for machinery, and commercial buildings are frequently rented out by their owners, it is more common still for businesses to own their capital. In this case, they can be thought of as renting the assets to themselves. But then there is no arms-length rental price to be observed. Even in the case of leased assets, it is generally easier to observe the asset price than the rental price.

It is therefore desirable to find an expression for the (usually unobserved) rental price in terms of the asset price, which can be observed more readily. Solving equation **(1)** for the rental price:

$$p^K_{t,0} = r_t \cdot p^A_{t-1,0} + (p^A_{t-1,0} - p^A_{t,1}) \qquad \textbf{(3)}$$

The second term on the right-hand side is the gain or loss from holding the asset for one period. Sometimes this second term is called 'depreciation', but this is not the sense in which that term is used here. Two factors affect the second term: first, the asset is now one year older, and second, time has moved on one period. It is useful to take separate account of these two factors by adding and subtracting the current price of a new machine, $p^A_{t,0}$, in the right-hand side of **(3)**:

$$p^K_{t,0} = r_t \cdot p^A_{t-1,0} + (p^A_{t,0} - p^A_{t,1}) - (p^A_{t,0} - p^A_{t-1,0}) \qquad \textbf{(4)}$$

Here the two bracketed terms on the right-hand side can be interpreted as

Depreciation: $(p^A_{t,0} - p^A_{t,1})$

Capital gain/loss: $(p^A_{t,0} - p^A_{t-1,0})$

Note that depreciation is measured as the difference between the prices of a new and a one year old asset *at a point in time t*, while the capital gain/loss is measured as the change in the price of a new asset *between periods t*-1 *and t*. Putting it another way, depreciation is a cross-section concept while capital gain/loss is a time series one, as is illustrated in the following matrix of new and second-hand asset prices:

	Period	
Age	*t*-1	*t*
0	$p^A_{t-1,0}$	$p^A_{t,0}$
1	$p^A_{t-1,1}$	$p^A_{t,1}$

Reading down the columns shows depreciation, while reading across the rows traces capital gains or losses. Defined in this way, it is quite reasonable to expect that even assets like London houses depreciate. In June 2002, the price of an 80 year old, four-bedroom terrace house in Islington may have been lower than that of a 70 year old house in Islington of comparable specification, even though the owners of both houses were hoping that their values would have risen by June 2003.

If we define the rate of depreciation during period *t* as $\delta_t = (p^A_{t,0} - p^A_{t,1}) / p^A_{t,0}$, then equation **(4)** becomes:

$$p^K_{t,0} = r_t \cdot p^A_{t-1,0} + \delta_t \cdot p^A_{t,0} - (p^A_{t,0} - p^A_{t-1,0}) \qquad \textbf{(5)}$$

which is the Hall-Jorgenson formula for the cost of capital in discrete time (Hall and Jorgenson (1967)). Equation **(5)** expresses the rental price in terms of the prices of *new* assets, the rate of return, and the depreciation rate. The prices of new assets are certainly observable; indeed they *must* be observed if we are to measure investment, and hence asset stocks, in constant prices.

Since, from now on, we will be dealing only with new asset prices, it is convenient to simplify the notation by dropping the age subscripts. But we also need to recognise explicitly that assets are of many different types. Let p^A_{it} be the price of a new asset of type *i* in period *t* and let p^K_{it} be the corresponding rental price. Then equation **(5)** can be rewritten as:

$$p^K_{it} = r_t \cdot p^A_{i,t-1} + \delta_i \cdot p^A_{it} - (p^A_{it} - p^A_{i,t-1}) \qquad \textbf{(6)}$$

In moving from **(5)** to **(6)** we have introduced two substantive economic assumptions, as well as a notational change. First, we are assuming that the rate of return r_t is the same on all types of asset (we write r_t rather than r_{it}). Second, we are assuming that the rate of depreciation on a new asset of a given type does not vary over time, so that we write δ_i, not δ_{it}. The first assumption is consistent with profit maximisation. Certainly, firms would like to equalise rates of return *ex ante*. But *ex post*, things might turn out differently if they are unable to adjust the size of their holdings with equal speed for all types of asset. For example, an airline may be able to adjust its stock of computers more easily than its stock of planes. The assumption of equal rates of return might be particularly hard to maintain in a recession and perhaps too in a boom characterised by 'irrational exuberance'.

The second assumption, that depreciation rates do not vary over time, is obviously not true in general. However, it is well supported as a rule of thumb by studies of second-hand asset prices (see below, Section 3). Our second assumption is much weaker than assuming geometric depreciation. But it turns out to be very convenient to assume geometric depreciation when constructing capital stocks (see below).

Notice that, to measure the value of the marginal product of capital, we do not need to ask why asset prices are changing, we just need to measure them. Also, we do not need to take a view as to the causes of depreciation. Is it due to obsolescence or to physical decay? At this point, it does not matter.

One adjustment is needed to **(6)**, to take account of taxes on profits and subsidies to investment. This can be done by introducing a tax-adjustment factor into **(6)**:

$$p_{it}^{K} = T_{it}\left[r_t \cdot p_{i,t-1}^{A} + \delta_i \cdot p_{it}^{A} - (p_{it}^{A} - p_{i,t-1}^{A})\right] \qquad \textbf{(7)}$$

Here r_t must now be interpreted as the post-tax rate of return and T_{it} is the tax-adjustment factor:

$$T_{it} = \left[\frac{1-u_t D_{it}}{1-u_t}\right]$$

where u_t is the corporation tax rate and D_{it} is the present value of depreciation allowances as a proportion of the price of assets of type i.

Aggregating over vintages[(4)]

Consider a production function where output (Y) depends on the amount of the different vintages of capital which still survive and on other inputs. For notational simplicity and without loss of generality, we assume for the moment just one type of capital and one type of labour (L). Then the production function at time t+1 can be written:

$$Y_{t+1} = f(I_t, I_{t-1}, ..., I_{t-n}; L_{t+1}) \qquad \textbf{(8)}$$

where I_{t-i} is that part of investment made i years ago that still survives and the oldest assets still surviving are assumed to be n years old. Assuming constant returns to scale, by Euler's Theorem:

(4) See Fisher (1965) for a general discussion of aggregation over vintages. Diewert and Lawrence (2000) compare straight-line, geometric and one-hoss shay patterns of depreciation and discuss how the pattern affects aggregation over vintages.

$$Y_{t+1} = f_0 \cdot I_t + f_1 \cdot I_{t-1} + ... + f_n \cdot I_{t-n} + f_{n+1} \cdot L_{t+1} \quad \textbf{(9)}$$

where $f_s = \partial f / \partial I_{t-s}$, is the marginal product of machines of age *s*, and $f_{n+1} = \partial f / \partial L_{t+1}$ denotes the marginal product of labour. Define the aggregate capital stock *A* as:

$$A_t = I_t + (f_1 / f_0) \cdot I_{t-1} + (f_2 / f_0) \cdot I_{t-2} + ... + (f_n / f_0) \cdot I_{t-n} \quad \textbf{(10)}$$

where each vintage is weighted by its marginal product relative to that of a new machine. The *services* (*K*) from this aggregate are assumed to be proportional to the stock at the end of the previous period (beginning of the current period):

$$K_{t+1} = A_t \quad \textbf{(11)}$$

where the constant of proportionality is normalised to unity. Equation **(10)** is a sensible definition of the aggregate stock since we can now rewrite **(9)** as:

$$Y_{t+1} = f_0 \cdot K_{t+1} + f_{n+1} \cdot L_{t+1} \quad \textbf{(12)}$$

In other words, the contribution of all the vintages of capital to output equals the marginal product of a new machine (f_0) times the volume of capital services, as defined in equations **(10)** and **(11)**.

Another way to look at the aggregate stock is the following. Past investments $I_t, I_{t-1}, ..., I_{t-n}$ are all measured in the same units.[5] So to calculate their capacity to produce output it is reasonable to add them up, after allowing for the fact that the capacity of earlier investments has decayed somewhat since installation. This is what equation **(10)** accomplishes.

Equation **(11)** seems to imply that we are assuming full utilisation of capital at all times. This is not the case. As Berndt and Fuss (1986) have shown, the degree of utilisation is under certain assumptions measured correctly by the weight attached to aggregate capital services (f_0 in equation **(12)**), rather than by adjusting the capital aggregate itself. For example, if capital is underutilised during a recession, then its marginal product will be low. But then the share of profits in total income will be low too. In fact, the profit share is pro cyclical, so variations in utilisation will be captured by movements in the share, at least to some extent.

Now define the decay factor $(1 - d_s) = f_s / f_0$, $s = 0, ..., n$, where d_s is the rate of decay experienced by machines *s* years old.[6] Then the aggregate capital stock is:

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(5) Investment in a given asset is measured in practice as the nominal value of investment deflated by a price index. The price index (eg a producer price index) in principle corrects for any quality change, so that in real terms investment is in units of constant quality. Of course, there is some doubt as to how accurately price indices do capture quality change (Gordon (1990)).

(6) The concept of decay employed here covers both 'output decay' and 'input decay' (Feldstein and Rothschild (1974); OECD (2001b)). Output decay occurs when, with unchanged inputs, the output from a given asset declines over time, eg as a result of mechanical wear and tear. 'Input decay' occurs when maintaining output requires increasing other inputs, eg rising maintenance expenditure.

$$A_t = \sum_{s=0}^{n} (1-d_s) I_{t-s} \quad \textbf{(13)}$$

Because rental prices measure marginal revenue products, there is a connection between them and the weights in the capital aggregate **(10)**:

$$p_{t,s}^K / p_{t,0}^K = f_s / f_o \quad \textbf{(14)}$$

A great simplification is achieved if we assume that the rate of decay is constant over time: $1-d_s = (1-d)^s, \forall s$. Here *d* is the geometric rate of decay. Then we have:

$$f_s / f_o = (1-d)^s \quad \textbf{(15)}$$

The equation for the capital stock now takes a particularly simple form. From **(13)**:

$$A_{it} = I_{it} + (1-d_i) A_{i,t-1} \quad \textbf{(16)}$$

where we have introduced an additional subscript *i* to indicate that this relationship applies to each of potentially many types of asset.

Depreciation and decay

What is the relationship between the rate of decay and the rate of depreciation? The former is a 'quantity' concept: the rate at which the services derivable from a capital asset decline as the asset ages. The latter is a 'price' concept: the rate at which the price of an asset declines as it ages. That these are not necessarily the same can be seen from the example of assets with a 'light bulb' or 'one-hoss shay'(7) pattern of service (constant over the service life and falling immediately to zero at its end). In this case, decay is zero right up to the moment of failure. But a cross section of the new and second-hand prices of this asset will show the price steadily declining with age. The reason is that, though the annual return on the asset may be unchanged, the older the asset, the fewer the years over which this return is expected to be enjoyed.

However, in the case of geometric decay, it can be shown that, though the two *concepts* are different, the two *rates* are equal:

$$d_i = \delta_i \quad \textbf{(17)}$$

In this case, and only in this case, the rate of depreciation equals the rate of decay.(8)

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(7) The 'wonderful one-hoss shay' (a type of horse-drawn carriage), celebrated in a poem by Oliver Wendell Holmes that is reproduced in OECD (2001b), yielded a constant flow of services before disintegrating on its 100th birthday.

(8) The proof comes from noting that the asset price equals the present value of the future stream of rentals: see equation **(1)**. If decay is geometric, then from **(14)** and **(15)** the rental price of an asset of age *s* in any period is $(1-d)^S$ times the price of a new asset in the same period. It follows that the corresponding asset prices must stand in the same ratio to each other. The converse is also true: if depreciation is geometric, then so is decay. See Appendix A for proof.

Aggregating over asset types

Let us say that we have solved the problem of how to aggregate over vintages of a given type of capital, but we still need to aggregate different asset types together. Suppose the true production function is given by:

$$Y_t = f(K_{1t}, K_{2t}, ..., K_{mt}; L_t, t) \qquad \textbf{(18)}$$

where there are m types of asset. We wish to replace this by a simpler function containing only aggregate capital services:

$$Y_t = g(K_t, L_t, t) \qquad \textbf{(19)}$$

The question is, what is the relationship between K_t and the individual K_{it}? Taking the total logarithmic derivative with respect to time in these two functions, we obtain:

$$\hat{Y}_t = \sum_{i=1}^{m} \left(\frac{\partial \ln Y_t}{\partial \ln K_{it}} \right) \cdot \hat{K}_{it} + \frac{\partial \ln Y_t}{\partial \ln L_t} \cdot \hat{L}_t + \frac{\partial \ln Y_t}{\partial \ln t}$$

$$\textbf{(20)}$$

$$\hat{Y}_t = \left(\frac{\partial \ln Y_t}{\partial \ln K_t} \right) \cdot \hat{K}_t + \frac{\partial \ln Y_t}{\partial \ln L_t} \cdot \hat{L}_t + \frac{\partial \ln Y_t}{\partial \ln t}$$

where a hat (^) denotes a growth rate, eg $\hat{Y}_t = d \ln Y_t / dt$. So for consistency we must have:

$$\hat{K}_t = \sum_{i=1}^{m} \left[\left(\frac{\partial \ln Y_t}{\partial \ln K_{it}} \right) \Big/ \left(\frac{\partial \ln Y_t}{\partial \ln K_t} \right) \right] \cdot \hat{K}_{it} \qquad \textbf{(21)}$$

The elasticities in **(21)** are not directly observable but, if inputs are paid the value of their marginal products, they can be equated with input shares:

$$\frac{\partial \ln Y_t}{\partial \ln K_{it}} = \frac{p_{it}^K K_{it}}{p_t Y_t}$$

$$\textbf{(22)}$$

$$\frac{\partial \ln Y_t}{\partial \ln K_t} = \frac{p_t^K K_t}{p_t Y_t}$$

where p_t is the output price and p_t^K is the rental price of aggregate capital (the value of the marginal product of aggregate capital), so $p_t^K K_t = \sum_{i=1}^{m} p_{it}^K K_{it}$ is aggregate profit. Consequently,

$$\hat{K}_t = \sum_{i=1}^{m} w_{it} \hat{K}_{it} \qquad \textbf{(23)}$$

where:

$$w_{it} = \frac{p_{it}^K K_{it}}{\sum_{i=1}^m p_{it}^K K_{it}}, \quad i = 1, \ldots, m \tag{24}$$

are the shares of each type of asset in aggregate profit. Equations **(23)** and **(24)** define the VICS in continuous time as a Divisia index. For empirical purposes, we need to define it in discrete time. The discrete time counterpart of a Divisia index is a chain index. Here we use a Törnqvist chain index:

$$\ln\left[K_t / K_{t-1}\right] = \sum_{i=1}^m \overline{w}_{it} \ln\left[K_{it} / K_{i,t-1}\right], \quad \overline{w}_{it} = (w_{it} + w_{i,t-1})/2 \tag{25}$$

An example

Suppose that the true production function of a competitive economy is:

$$Y_t = H_t \cdot K_{1t}^{\alpha} \cdot K_{2t}^{\beta} \cdot L_t^{1-\alpha-\beta}, \qquad H_t > 0$$

where there are two types of capital. Suppose we wish to use a capital aggregate K rather than distinguish the two types. We know that the share of profit in national income is $\alpha + \beta$, so it is natural to write

$$Y_t = H_t \cdot K_t^{\alpha+\beta} \cdot L_t^{1-\alpha-\beta}$$

as the simplified production function. So for consistency we must have

$$K_t^{\alpha+\beta} = K_{1t}^{\alpha} \cdot K_{2t}^{\beta}$$

whence:

$$\hat{K}_t = \left(\frac{\alpha}{\alpha+\beta}\right) \cdot \hat{K}_{1t} + \left(\frac{\beta}{\alpha+\beta}\right) \cdot \hat{K}_{2t}$$

Here $\alpha/(\alpha+\beta)$, $\beta/(\alpha+\beta)$ can be interpreted as the shares of aggregate profit attributable to the two types of capital. This equation shows how to construct the VICS for this economy.

From theory to measurement

To calculate capital services from a particular type of asset, we need to estimate capital stocks (equation **(11)**). To calculate capital stocks, we need a back history of investment and we need to know the rates of decay (equation **(10)**). Decay rates are related to the rental prices of assets of different ages (equation **(14)**). Rental prices are normally unobserved but are related to asset prices (equation **(7)**). To estimate rental prices from equation **(7)**, we need to know also depreciation rates and the rate of return. Having estimated capital stocks, we need rental prices again to weight together the services from different assets. Depreciation rates can in principle be found by econometric analysis of a panel of new and second-hand asset prices, following the methods of Hulten and Wykoff (1981a) and (1981b) for example (see Section 3 below). To apply this approach to all types of assets would constitute a very ambitious programme of empirical research, which has not been carried out in its full entirety anywhere in the world (see Section 3 again for more on this).

The problem of estimating wealth and VICS measures on a consistent basis can be greatly simplified (both from a theoretical and an empirical point of view) if we follow Jorgenson and his various collaborators (eg Jorgenson *et al* (1987); Jorgenson and Stiroh (2000)) and assume geometric depreciation and consequently also geometric decay. Under the geometric assumption, the equations of the model (**(7)**, **(11)**, **(16)**, **(24)** and **(25)**) simplify to the following:

$$A_{it} = I_{it} + (1-\delta_i)A_{i,t-1} \qquad \textbf{(26)}$$

$$K_{it} = A_{i,t-1} \qquad \textbf{(27)}$$

$$p_{it}^K = T_{it}\left[r_t \cdot p_{i,t-1}^A + \delta_i \cdot p_{it}^A - (p_{it}^A - p_{i,t-1}^A)\right] \qquad \textbf{(28)}$$

$$\begin{aligned} &\ln\left[K_t / K_{t-1}\right] = \sum_{i=1}^m \bar{w}_{it} \ln\left[K_{it} / K_{i,t-1}\right], \\ &\bar{w}_{it} = (w_{it} + w_{i,t-1})/2,\ w_{it} = \frac{p_{it}^K K_{it}}{\sum_{i=1}^m p_{it}^K K_{it}},\ \ i = 1,\dots,m \end{aligned} \qquad \textbf{(29)}$$

Empirically, this is a considerable simplification. It is assumed that we have the investment series I_{it}, the tax adjustment factors T_{it}, and the asset prices p_{it}^A. Provided we also know the depreciation rates δ_i on each asset, we can now estimate the stocks. To calculate the rental prices we need to know the rate of return too. But we can estimate this from the fact that observed, aggregate profits (Π), that is, gross operating surplus before corporation tax and depreciation, must equal the total rentals generated by all the assets:

$$\Pi_t = \sum_{i=1}^m p_{it}^K K_{it} = \sum_{i=1}^m T_{it} \cdot \left[r_t \cdot p_{i,t-1}^A + \delta_i \cdot p_{it}^A - (p_{it}^A - p_{i,t-1}^A)\right] \cdot K_{it} \qquad \textbf{(30)}$$

This equation contains only one unknown, r_t, so we can rearrange it to solve for the unknown rate of return. Economically, this means that we are interpreting r_t as the actual, realised, post-tax rate of return. Now we can calculate the rental prices and hence the VICS.

The model just set out is reasonable as long as the period is short (say quarterly). But if applied to annual data it is subject to two criticisms. First, the first equation states that investment done in period t is not subject to depreciation until the subsequent period. This is equivalent to assuming that investment is done at the end of the period. So if a computer is in reality purchased on 1 January 2001 the model says that it only starts depreciating on 1 January 2002. Second, capital services are assumed proportional to the stock at the end of the previous period. So a computer purchased on 1 January 2001 yields no services till 1 January 2002. Both these features are unrealistic. A slightly more complex model, which assumes that investment is spread evenly over the year and capital services are proportional to the stock at the midpoint of the year, is more appropriate for annual data. The equations of this model are as follows:

$$B_{it} = I_{it} + (1-\delta_i)\cdot B_{i,t-1}, \qquad i = 1,\ldots,m \tag{31}$$

$$A_{it} = (1-\delta_i/2)\cdot B_{it} \tag{32}$$

$$K_{it} = \overline{A}_{it} = \left[A_{i,t-1}\cdot A_{it}\right]^{1/2}, \qquad i = 1,\ldots,m \tag{33}$$

$$p_{it}^{K} = T_{it}\left[r_t\cdot p_{i,t-1}^{A} + \delta_i\cdot p_{it}^{A} - (p_{it}^{A} - p_{i,t-1}^{A})\right], \qquad i = 1,\ldots,m \tag{34}$$

$$\Pi_t = \sum\nolimits_{i=1}^{m} p_{it}^{K} K_{it} = \sum\nolimits_{i=1}^{m} T_{it}\cdot\left[r_t\cdot p_{i,t-1}^{A} + \delta_i\cdot p_{it}^{A} - (p_{it}^{A} - p_{i,t-1}^{A})\right]\cdot K_{it} \tag{35}$$

$$\begin{aligned}&\ln\left[K_t/K_{t-1}\right] = \sum\nolimits_{i=1}^{m}\overline{w}_{it}\ln\left[K_{it}/K_{i,t-1}\right],\\ &\qquad \overline{w}_{it} = (w_{it}+w_{i,t-1})/2,\ w_{it} = \frac{p_{it}^{K}K_{it}}{\sum\nolimits_{i=1}^{m}p_{it}^{K}K_{it}},\quad i = 1,\ldots,m\end{aligned} \tag{36}$$

$$\begin{aligned}&\ln\left[\overline{A}_t/\overline{A}_{t-1}\right] = \sum\nolimits_{i=1}^{m}\overline{v}_{it}\ln\left[\overline{A}_{it}/\overline{A}_{i,t-1}\right],\\ &\qquad \overline{v}_{it} = (v_{it}+v_{i,t-1})/2,\ v_{it} = \frac{p_{it}^{A}A_{it}}{\sum\nolimits_{i=1}^{m}p_{it}^{A}A_{it}},\quad i = 1,\ldots,m\end{aligned} \tag{37}$$

where:

m is the number of assets
A_{it} is the real stock of the ith type of asset at the *end* of period t
$\overline{A}_{it}$ is the real stock of the ith type of asset in the *middle* of period t
B_{it} is the real stock of the ith type of asset at the *end* of period t, if investment were assumed to be done at the end of the period, instead of being spread evenly through the period
K_{it} is real capital services from assets of type i during period t
I_{it} is real gross investment in assets of type i during period t
δ_i is the geometric rate of depreciation on assets of type i
r_t is the nominal post-tax rate of return on capital during period t
T_{it} is the tax-adjustment factor in the Hall-Jorgenson cost of capital formula
p_{it}^{K} is the rental price of new assets of type i, payable at the end of period t
p_{it}^{A} is the corresponding asset price at the end of period t
Π_t is aggregate profit (= nominal aggregate capital services) in period t
K_t is real aggregate capital services during period t
A_t is aggregate real wealth at the end of period t
$\overline{A}_t$ is aggregate real wealth in the middle of period t

Equations **(31)** and **(32)** describe the evolution of asset stocks. They can be shown to arise from the following accumulation equation:

$$A_{it} = (1-\delta_i/2)\cdot I_{it} + (1-\delta_i/2)\cdot(1-\delta_i)\cdot I_{i,t-1} + (1-\delta_i/2)\cdot(1-\delta_i)^2\cdot I_{i,t-2}^{2} + \ldots \tag{38}$$

The factor $(1-\delta_i/2)$ arises as investment is assumed to be spread evenly throughout the unit period, so on average it attracts depreciation at a rate equal to half the per-period rate. This assumption affects the level, but not the growth rate, of the capital stock.[(9)]

Equation **(33)** states that capital services *during* period *t* derive from assets in place in the *middle* of period *t*. The capital stock in the middle of period *t* is estimated as the geometric mean of the stocks at the beginning and end of the period. Equation **(34)** defines the rental price of assets of type *i*. Equation **(35)** says that aggregate profits are equal to the sum over all assets of the rental price times the asset stock. Equation **(36)** defines the growth rate of the VICS and equation **(37)** the growth rate of the wealth measure.

Equations **(36)** and **(37)** are chain indices of the Törnqvist type. It would also be possible to derive growth rates of the VICS and of real wealth using fixed weights, eg those of 1995, as currently in the National Accounts. Note, however, that the ONS is planning to move to annual chain-linking in 2003.

In our empirical work, we use both models. The quarterly model uses equations **(26)-(30)**, the annual model equations **(28)-(34)**. However, at a quarterly frequency we find the estimated rental prices to be unrealistically volatile. So we use the annual model to estimate the rental prices and we employ these for quarterly, as well as for annual data.

These models assume constant rates of depreciation over time. In our empirical work we deviate from this in one respect, since we have made an allowance for accelerated scrapping during recessions. We describe this more fully in Section 4.

Wealth measures of capital versus the VICS

How does the growth of a VICS compare with the growth of a wealth measure of capital? We answer this question using the simpler quarterly model of the previous subsection. Assuming geometric depreciation, the nominal value of capital (*W*) in a balance sheet sense at the beginning of period *t* (end of period *t*-1) is:

$$W_{t-1} = \sum_{i=1}^{m} p_{i,t-1}^{A} A_{i,t-1} = \sum_{i=1}^{m} p_{i,t-1}^{A} K_{it}$$

We can define a Törnqvist index of the growth of the aggregate real stock of capital (*A*) in the wealth sense as:[(10)]

$$\ln\left[A_{t-1}/A_{t-2}\right] = (1/2)\sum_{i=1}^{m}\left(v_{i,t-1}+v_{i,t-2}\right)\cdot\ln\left[A_{i,t-1}/A_{i,t-2}\right]$$

$$= (1/2)\sum_{i=1}^{m}\left(v_{i,t-1}+v_{i,t-2}\right)\cdot\ln\left[K_{it}/K_{i,t-1}\right]$$

(9) This assumption corresponds to the practice of the BEA: see U.S. Department of Commerce (1999, box on page M-5).

(10) A similar index of the wealth stock is published by the BEA (Herman (2000)). Their index is Fisher rather than Törnqvist but in practice these two types of chain index yield very similar results.

where the v_{it} are the shares of each asset in the nominal value of the capital stock (V):

$$v_{i,t-1} = p^{A}_{i,t-1}A_{i,t-1} / \sum_{i=1}^{m} p^{A}_{i,t-1}A_{i,t-1}$$

The growth rate of the VICS (see equation **(29)**) is:

$$\ln[K_t / K_{t-1}] = (1/2)\sum_{i=1}^{m}(w_{it} + w_{i,t-1}) \cdot \ln[K_{it} / K_{i,t-1}]$$

The only difference between the growth rates of wealth and the VICS is the weights, $v_{i,t-1}$ instead of w_{it}. The wealth measure uses asset prices in the weights while the VICS uses rental prices, these prices being related by equation **(28)**. It is clear then that the higher the ratio of the rental to the asset price, the larger the weight that an asset will receive in the VICS. Intuitively, it is clear that if an asset has a higher-than-average rental price in proportion to its asset price, then its VICS weight will be higher than its wealth weight. This is proved formally in Appendix A.

If it turns out that the stocks of those assets with high rental price to asset price ratios tend to grow more rapidly, then a VICS will grow more rapidly than a wealth measure. Empirically, this has indeed been the case in recent decades. The service life of plant and machinery is short relative to that of buildings, hence their rental price is relative higher. And stocks of plant and machinery have grown more rapidly than those of buildings. The difference between asset and rental price weights is particularly large for assets like computers. Not only is their service life very short but their prices have been falling, ie holding them incurs a capital loss. So their rental price has to be very high (around 60% of the asset price) to make them profitable. Within the plant and machinery category, stocks of computers have been growing exceptionally rapidly (Oulton (2001a)).

In addition to the growth rates, we can if desired also derive the levels of real wealth and the VICS. In the case of real wealth, we can take the level of *nominal* wealth in some base period s (eg 1995):

$$\sum_{i=1}^{m} p^{A}_{is}A_{is}$$

and generate a series in 'chained 1995 pounds' by applying to this expression the growth rates given by equation **(37).** Note though that this will *not* yield the same result as would come from calculating the stock of each asset in period s prices and then adding the individual stocks. The reason is that the components of a chain index do not in general add to the chained total. Similarly, we can generate a series for the real level of the VICS by applying the growth rates given by equation **(36)** to the nominal level in base period s, which is just the level of profits in that period, Π_s : recall that the VICS measures the flow of capital services. Note that if we now compare the level of the VICS with the level of wealth, we are comparing a flow with a stock. This may be legitimate, but care should be taken over the interpretation: comparisons between the absolute size of the two measures are not meaningful.

3 Depreciation and replacement

The concepts of depreciation and replacement are related but distinct. Depreciation relates to the wealth measure of capital, replacement to the (misnamed) 'productive' capital stock, otherwise (and better) known as the VICS. Aggregate depreciation is the fall in the value of the capital stock which would occur if gross investment were zero. Alternatively, it is the amount of investment necessary to maintain the value of the stock at its current level. Replacement is the amount of investment necessary to maintain *the flow of capital services* at its current level. The difference between the two concepts is clearest in the case where the productive capacity of an asset follows the 'light bulb' pattern, ie constant up till the moment of failure. In this case, the asset falls in value with age, since there are progressively fewer years over which profits can be earned. But replacement is zero up till the moment of failure. Suppose the asset in question lasts for ten years and all investment has taken place in the last eight years. Then in the current year replacement is zero, since at the end of the year the oldest asset will be nine years old and will still be yielding the same flow of service as it did when new. But total depreciation will be positive since the assets are approaching the end of their lives. The discounted flow of future profits is falling as the assets age, so their value is declining even though their productive efficiency is unchanged.

Depreciation is also called capital consumption by national income statisticians. If depreciation is subtracted from gross investment, the result is usually called net investment. But if depreciation and replacement are not the same, then net investment so defined does not equal the increase in the VICS. There is one case, however, where aggregate depreciation and aggregate replacement are equal in value, namely when depreciation is geometric.

Consider a single, homogeneous asset, so that for the moment we ignore issues of quality adjustment. Measured in current prices, the value of the wealth stock of this asset at the end of period *t*, W_t, is:

$$W_t = p^A_{t,0}\phi_{t,0}I_t + p^A_{t,1}\phi_{t,1}I_{t-1} + p^A_{t,2}\phi_{t,2}I_{t-2} + \dots \qquad \textbf{(39)}$$

where:

$p^A_{t,s}$ is the price at time *t* of an asset which is aged *s* at *t*

$\phi_{t,s}$ is the proportion of assets of age *s* which survive at time *t* and we set $\phi_{t,0} = 1$

and I_{t-s} is the volume of investment in this asset which was carried out in period $t-s$ (the number of machines installed in *t-s*)

In period *t* prices, the value of wealth in the *previous* period, *t*-1, is:

$$W_{t-1} = p^A_{t,0}\phi_{t-1,0}I_{t-1} + p^A_{t,1}\phi_{t-1,1}I_{t-2} + p^A_{t,2}\phi_{t-1,2}I_{t-3} + \dots$$

The relationship between wealth today and wealth yesterday, measured in *today's* prices, is:

$$W_t = p^A_{t,0}I_t - D_t + W_{t-1}$$

where D is depreciation in period t prices. Hence:

$$D_t = p^A_{t,0}[(\phi_{t-1,0} - (p^A_{t,1}/p^A_{t,0})\phi_{t,1}]I_{t-1} + [(p^A_{t,1}/p^A_{t,0})\phi_{t-1,1} - (p^A_{t,2}/p^A_{t,0})\phi_{t,2}]I_{t-2} + ... \qquad \textbf{(40)}$$

Note that the prices here are all dated to period t: they are the new and second-hand prices of this asset at time t, weighted by the probability of survival.

Now consider replacement investment for this asset. Assume that capital services during time t derive from assets installed in period t-1 and earlier. An asset installed in period t-s yields a flow of services during t which we denote by $p^K_{t,s}$. As in the previous section, we can think of $p^K_{t,s}$ as the *rental* price of an asset of this vintage. Define K_t as the value of total capital services from the stock of this asset at time t in period t prices. Then we have:

$$K_{t+1} = p^K_{t,0}\phi_{t,0}I_t + p^K_{t,1}\phi_{t,1}I_{t-1} + ...$$

The services that require replacement investment during t, R'_t, measured again in the prices of period t, are defined implicitly by:

$$K_{t+1} = p^K_{t,0}I_t - R'_t + K_t$$

whence:

$$R'_t = p^K_{t,0}[(\phi_{t-1,0} - (p^K_{t,1}/p^K_{t,0})\phi_{t,1}]I_{t-1} + [(p^K_{t,1}/p^K_{t,0})\phi_{t-1,1} - (p^K_{t,2}/p^K_{t,0})\phi_{t,2}]I_{t-2} + ...$$

This gives the value of the *services* that need require to be replaced: this is the reduction in the value of services which would occur if gross investment were zero. The value of *investment* needed for replacement is therefore:

$$\begin{aligned} R_t &= R'_t(p^A_{t,0}/p^K_{t,0}) \\ &= p^A_{t,0}[(\phi_{t-1,0} - (p^K_{t,1}/p^K_{t,0})\phi_{t,1}]I_{t-1} + [(p^K_{t,1}/p^K_{t,0})\phi_{t-1,1} - (p^K_{t,2}/p^K_{t,0})\phi_{t,2}]I_{t-2} + ... \end{aligned} \qquad \textbf{(41)}$$

Comparing the equations for depreciation and replacement, **(40)** and **(41)**, we see that there is no reason in general to expect the two measures to be the same. The one involves asset prices, the other rental prices. However, if depreciation is geometric, then we can show that they will in fact be equal. With geometric depreciation, both the asset price and the rental price decline with age at the rate of depreciation (δ):

$$p^A_{t,s}/p^A_{t,0} = p^K_{t,s}/p^K_{t,0} = (1-\delta)^s, \quad s = 0,1,2,...$$

Hence in this case we see from **(40)** and **(41)** that:

$$R_t = D_t \qquad \textbf{(42)}$$

Also the asset accumulation equation in current prices **(39)** now takes the simple form:

$$W_t = p^A_{t,0}I_t + (1-\delta)W_{t-1}$$

The aggregate depreciation rate

There is frequent interest in the aggregate depreciation rate. A rising aggregate rate suggests that the mix of assets in the capital stock is shifting towards assets with shorter lives. In turn, this implies that a given amount of gross investment will lead to lower growth in both the wealth stock and the VICS than if the mix were not changing. But we can measure the aggregate depreciation rate in either nominal terms or real terms and these measures may behave in quite different ways. In nominal terms, the aggregate rate is the ratio of aggregate nominal depreciation to aggregate nominal wealth (the nominal capital stock). In real terms, there are two measures. Either we can take the ratio of aggregate real depreciation to aggregate real wealth or we can back out the depreciation rate from the aggregate capital accumulation equation. In this subsection we show that the nominal definition has a natural interpretation. But both the real definitions can produce counterintuitive results. For example, under chain-linking the aggregate real rate can rise without limit so that it eventually exceeds the rate on any individual asset. Therefore we should exercise caution when using the real definitions.

In nominal terms, the aggregate depreciation rate is the ratio of aggregate nominal depreciation to aggregate nominal wealth (the nominal capital stock):

$$\delta_t^N = \frac{\sum_{i=1}^{m} \delta_i p_{it} A_{i,t-1}}{\sum_{i=1}^{m} p_{it} A_{i,t-1}} \qquad \textbf{(43)}$$

where δ_i is the depreciation rate on the ith asset, A_{it} is the stock of the ith asset at the *end* of period *t*, and p_{it} is the corresponding asset price.[(11)] One definition of the real rate is the ratio of aggregate *real* depreciation to aggregate *real* wealth:

$$\delta_t^R = D_t / A_{t-1} \qquad \textbf{(44)}$$

where D_t is aggregate real depreciation and A_t is the aggregate real capital stock at the end of period *t*.

A second definition of the real rate is the rate that can be backed out from the aggregate capital accumulation equation:

$$A_t = I_t + (1 - \delta_t^B) A_{t-1}$$

whence:

$$\delta_t^B = (I_t / A_{t-1}) + [(A_t - A_{t-1}) / A_{t-1}] \qquad \textbf{(45)}$$

where I_t is aggregate real gross investment.

(11) This definition assumes geometric depreciation, which is used below. But the nominal definition could of course be extended to the non-geometric case.

If these three measures were calculated for a single asset, the results would be identical. But when done at the aggregate level the results will differ.

Comparing the measures

(a) The nominal measure, δ_t^N

One reason for considering the nominal measure is that it squares well with the way depreciation is estimated by the BEA. In the US national accounts, the stock of any asset is assumed to evolve (approximately) according to the simple accumulation equation:

$$A_{it} = I_{it} + (1-\delta_i)\cdot A_{i,t-1}$$

where A_{it} is the stock of the ith asset at the *end* of period *t*, I_{it} is gross investment in period *t* and δ_i is the depreciation rate. With a few exceptions, the individual δ_i are not assumed to change over time.(12) So any change in the ratio of aggregate depreciation to the aggregate capital stock (in current prices) indicates a change in the asset composition of the capital stock. That is,

$$\delta_t^N = \frac{\sum_{i=1}^m \delta_i p_{it} A_{i,t-1}}{\sum_{i=1}^m p_{it} A_{i,t-1}} = \sum_{i=1}^m \left(\frac{p_{it} A_{i,t-1}}{\sum_{i=1}^m p_{it} A_{i,t-1}} \right) \cdot \delta_i$$

In other words, the aggregate depreciation rate is a weighted average of the rates on individual assets, where the weights are the shares of each asset in aggregate wealth. So the aggregate rate must necessarily be bounded by the rates on the individual assets.

(b) The first real measure, δ_t^R

Let us compare the nominal measure with the first of the two real measures, δ_t^R. We will show that, under chain-linking, the latter can produce unacceptable results. Consider a simple case where there are two assets, one with a high depreciation rate, the other with a low one. Assume that the real stock of the high depreciation asset is growing more rapidly than that of the low depreciation one (which is the case at the moment for computers and software). Suppose that the share of each asset in the value of wealth is constant over time (the Cobb-Douglas case). If the importance of the assets, as measured by wealth shares, is not changing, and the individual depreciation rates are constant, then it seems reasonable that the aggregate rate should be constant too. And this will certainly be true of the aggregate depreciation rate in nominal terms. However, it can be shown that the aggregate rate defined in real terms will rise without limit in this case. Eventually, it will be higher than either of the two individual rates! This is not a reasonable way for a measure of the aggregate rate to behave. So though we might go on using such a measure for modelling purposes, we cannot expect it necessarily to behave in ways consistent with our economic intuition.

The intuition behind this result is as follows. The growth rate of real depreciation, like the growth rate of real wealth, is a weighted average of the growth rates of the components. It can be shown that the weight on the high-depreciation asset in the depreciation index is larger than its weight in the wealth index. Consequently, if the high-depreciation asset is growing more rapidly, then real

(12) See Fraumeni (1997) and U.S. Department of Commerce (1999).

depreciation will grow more rapidly than real wealth. It follows that the ratio of real depreciation to real wealth will rise over time without limit, even if all individual depreciation rates and the shares of each asset in total wealth are constant over time (see Appendix A for a proof).

This result holds under chain-linking. With a fixed-base index, the aggregate real rate will approach the higher of the two individual rates asymptotically (see the Appendix again). In the US National Income and Product Accounts (NIPA), the growth of real depreciation, like the growth of the wealth stock, is calculated as a chain index (annual chain-linking). So this result certainly applies to the United States. In the United Kingdom, the weights are updated about every five years ('quinquennial chain-linking'). So over long periods, but not short ones, the result applies to the United Kingdom too.

(c) The second real measure, δ_t^B

Whelan (2000b) has proved a related but different result about the second real measure. Suppose we calculate the aggregate depreciation rate by backing it out from the aggregate accumulation equation:

$$\delta_t^B = (I_t / A_{t-1}) - [(A_t - A_{t-1}) / A_{t-1}]$$

which is equation **(45)**. Suppose that there are again two assets but now with the same depreciation rate. Assume that asset 1 is growing more rapidly than asset 2 but that wealth shares are constant. Clearly in this situation the aggregate depreciation rate is constant. But Whelan shows that the backed out rate δ_t^B derived from aggregate data will rise without limit. The explanation is that the weight of asset 1 in investment is higher than its weight in wealth, so investment is growing more rapidly than wealth and the I_t / K_{t-1} ratio is trending upwards.

Conclusion on aggregate depreciation measures

Measuring the aggregate depreciation rate in real terms, by either method, can lead to serious problems. So measured, the aggregate rate can be higher (or lower) than any of the rates on individual assets. And the aggregate rate can trend upwards (or downwards) without limit even though nothing is really happening in economic terms. There are certainly signs in the US data that this is not just a theoretical possibility. The upward drift in the aggregate rate is much less in nominal than in real terms. This suggests that we should use real definitions with caution. We cannot expect them necessarily to behave in ways that are consistent with our economic intuition.

Straight-line as an alternative to geometric depreciation

In the US NIPA, depreciation is assumed to be (in most cases) geometric (Fraumeni (1997)). In the United Kingdom by contrast, the ONS (along with many other national statistical agencies) assumes that assets depreciate on a straight-line basis over their assumed asset life; retirement or scrapping is assumed to be normally distributed around the mean life. Accordingly, the purpose of this subsection is to compare the implications of geometric as opposed to straight-line depreciation for the depreciation rate of a given type of asset.

The overall rate of depreciation of the stock of some asset, or the rate of deterioration of the flow of capital services which it yields, arises from two factors. First, the retirement or scrapping of assets and second, the decline in efficiency of surviving assets. We consider each of these factors in turn, first for geometric and then for straight line.

Asset mortality: geometric assumption
Suppose that in some given year a number of machines of a particular type are added to the capital stock. We refer to these machines as a cohort. Suppose that there is a fixed probability of 'death' attached to this asset type and that this probability is independent of age. 'Death' might mean loss due to accidents, fires or explosions or it might mean voluntary scrapping for any reason. Denote the probability of death by m. Then the probability that a new example of this asset lives for t years is:

$$(1-m)^t m$$

This is a geometric distribution, so the expected life n of a new asset is:

$$n = (1-m)/m \approx 1/m \text{ for small } m$$

(See eg Feller (1968), Vol. 1, page 268.) Hence the mortality rate m is related to the mean life as:

$$m = 1/(n+1)$$

We can also ask: what proportion of the original cohort is expected to survive after L years? This is given by:

$$(1-m)^L = [1-1/(n+1)]^L$$

For n = 5, 10, 20, or 30 we get 40%, 39%, 38% and 37% respectively, substantially less than 50%. This is not surprising. The period of time after which only 50% of the original cohort survives is the *median* life. With a distribution skewed to the right, as is this one, the median life is less than the mean life. In fact for this distribution, the median life is about 70% of the mean life:

Table A Mean and median life lengths under geometric depreciation (years)

Mean	*Median*
5	3.8
10	7.3
20	14.2
30	21.1
40	28.1

Note: The median life n is calculated as the solution to $[1-(1/n+1)]^n = 0.5$ ie $n = \ln(0.5)/\ln[1-1/(n+1)]$.

Declining efficiency with age
According to basic capital theory, the price of an asset is the present value of the services it is expected to yield over its remaining life (see Section 2 above). 'Services' refers to the marginal

revenue product of the asset. If assets could be hired, then the rental price would equal the asset's marginal revenue product, in the same way that the wage equals the marginal revenue product of labour. If assets are not expected to last forever, then older assets will command a lower price than newer ones at any point in time, simply because the stream of future services is expected to be shorter. This effect is already accounted for in the discussion of mortality. But in addition, there is the possibility that the services of a surviving asset decline with age. This gives an additional reason for the asset price to decline with age and also must be accounted for if we want to measure capital services correctly. For present purposes, we do not need to discuss why an asset's services might decline with age, just to examine the consequences if this is indeed occurring.

The standard way this has been dealt with in practice in the United States is to assume that depreciation is 'accelerated' by comparison with what would occur if scrapping were the only force at work. The depreciation rate is expressed as:

$$\delta = R / n$$

where R is termed the 'declining balance rate'. In the past, R = 2 was frequently chosen; this is referred to as the 'double declining balance' method. This implies that the efficiency of surviving assets declines at the constant rate $1/n$ while separately and independently the force of mortality is $1/n$ too: that is, $(1-\delta) = (1-1/n)\cdot(1-1/n) \approx 1-2/n$. In the US NIPA a variety of values of R are now employed, based on the Hulten-Wykoff studies. Typically, R = 1.65 for equipment and R = 0.91 for private non-residential structures (Fraumeni (1997)).

Straight-line depreciation

Under straight-line depreciation, the gross stock (GA) of asset i at the end of period t is the cumulated sum of all surviving vintages of investment:

$$GA_{it} = \sum_{s=0}^{n_i-1} I_{i,t-s}$$

where I_{it} is gross investment in asset i during period t and n_i is the asset's life. Under the assumption of straight-line depreciation, an asset loses a fraction $1/n_i$ of its *initial* value in each period. Since assets of each surviving vintage depreciate by an equal amount per period, overall depreciation (capital consumption) on asset i in period t, D_{it}, is:

$$D_{it} = GA_{it} / n_i$$

The net stock of asset i at the end of period t is the gross stock less cumulated depreciation:

$$A_{it} = GA_{it} - \left[\sum_{s=0}^{n_i-1} sI_{i,t-s} / L_i\right]$$

The depreciation *rate* is defined as depreciation in period t as a proportion of the *net* stock of the asset at the end of the previous period:

$$\delta_{it} = D_{it} / A_{i,t-1}$$

An important point to note is that the straight-line depreciation rate in general varies over time, even when asset life is assumed constant. The rate is in fact a function of the age structure of the stock: the younger the stock, the lower the rate. Suppose that the stock consists entirely of the oldest surviving vintage, there having been no subsequent investment. Then the gross stock is $I_{i,t-L+1}$, the net stock is $I_{i,t-n_i+1}/n_i$, and depreciation is $I_{i,t-n_i+1}/n_i$. So the depreciation rate is 1. On the other hand, suppose the gross stock consists entirely of investment done in the last period, $I_{i,t-1}$. Then depreciation is $I_{i,t-1}/n_i$ and the depreciation rate is $1/n_i$. So in general, the depreciation rate varies between $1/n_i$ and 1. If an investment boom occurs, then other things equal the depreciation rate falls.

Geometric versus straight-line depreciation

Suppose that an asset costs £1 when new in year *t*. If depreciation is geometric, then the actual nominal value of depreciation (or capital consumption) on an asset aged *s* years in year *t* is

$$\delta(1-\delta)^s$$

Clearly, capital consumption itself declines geometrically as the asset ages (*s* rises). It is highest in the asset's first year and approaches zero asymptotically as the asset ages. But by definition the depreciation *rate* (depreciation as a proportion of the asset's price) is constant.

With straight-line depreciation, capital consumption on a particular asset type is the same at each age, equal to a fraction 1/*n* of the price when new. So depreciation as a proportion of the second-hand asset price, the depreciation rate, is rising as the asset ages. The depreciation rate for an asset of age *s* is:

$$\frac{1}{n-(s-1)}$$

This equals (100/*n*)% in the first year of life and 100% in the last year of life. It follows that the total depreciation rate on a particular asset class depends on the age structure of the stock. Under geometric depreciation by contrast, total depreciation is independent of the age structure.

Because the price of an asset is the present value of the services it is expected to yield over its remaining life, there is a connection between depreciation and the rate at which services are changing (decay). If depreciation is geometric, then decay is geometric too and at the same rate. So, under geometric depreciation, old assets never apparently die but just fade away. This is best understood in a probabilistic sense: individual members of a cohort die, but the cohort as a whole goes on forever, though eventually its size is insignificant. Under straight-line depreciation, it can be shown that services decline linearly with age and then fall instantaneously to zero at the end of the asset's finite life. The main problem with straight-line depreciation is that it does not fit the facts. Empirical studies show that the age-asset price profile is generally convex, which is consistent with geometric depreciation. Under straight-line depreciation, the asset price should decline with age in a linear fashion.(13)

(13) See Oulton (2001b) for more on this.

Quantitative comparison between straight-line and geometric depreciation is not straightforward, since in the former the depreciation rate depends on the age structure of the stock. But a useful benchmark is provided for the case where investment is growing at a constant rate over the assumed life of the asset (a steady state). Table B below illustrates for a variety of asset-life lengths and growth rates of investment. The straight-line rate declines as the growth rate rises, since this shifts the age structure towards more recent vintages. But the effect is not very marked, except for the longest lives and high growth rates (which are in any case unlikely for long-lived assets). A five-year life corresponds in steady state to a geometric growth rate of about 30%-33% and a 20-year life to a geometric rate of 6%-9% (8%-9% if growth does not exceed 5% per annum).

In the United States, the depreciation rate for plant and machinery (excluding computers and software) averages about 13%. In the United Kingdom, the ONS assumes that plant and machinery has a life of 25-30 years in most industries. Table B shows that this is equivalent to a geometric rate of 5%-9% if growth does not exceed 5% per annum, much lower than the US rate.

Table B Steady-state depreciation rates when depreciation is straight line

	Growth rate of investment (% per annum)			
Life (years)	*2*	*5*	*10*	*20*
5	32.90	32.28	31.35	29.77
10	17.66	16.95	15.94	14.44
20	8.96	8.27	7.41	6.44
30	5.89	5.25	4.58	3.98
80	1.97	1.62	1.43	1.33

How do the levels of asset stocks and depreciation vary with the depreciation rate?
In a steady state, the *growth rate* of the stock of an asset is constant and equal to the growth rate of the investment which generates the stock. This is true whether depreciation is straight-line or geometric. But the depreciation rate does affect the *level* of the stock. Assuming a steady state, from the basic capital accumulation equation we can find that the end-year stock (A) is related to the investment flow by:

$$\frac{A_t}{I_t} = \left[\frac{1+g}{g+\delta}\right]$$

where g is the steady-state growth rate. The steady-state level of depreciation, as a proportion of gross investment, is therefore given by:

$$\frac{\delta A_{t-1}}{I_t} = \left[\frac{\delta}{g+\delta}\right]$$

These ratios are shown in Tables C and D. We see that the stock/gross investment ratio falls as the depreciation rate rises. The ratio is also negatively related to the growth rate. On the other hand, the depreciation/gross investment ratio rises with the depreciation rate. This is relevant when considering how the depreciation/GDP ratio might be expected to behave (though here aggregation issues and relative prices will also play a role). For example, if we change our estimate of the depreciation rate from 10% to 15%, then (for growth rates not exceeding 5%) we will lower the level of the asset stock by between 25% and 29%, while raising depreciation by 6%-12%.

Depreciation as a proportion of gross investment is high when growth rates are low and when depreciation rates are high. For computers, where the stock might grow at 20% per annum and the depreciation rate is 30%, the proportion would be 60% in steady state.

Table C Ratio of asset stock to investment in steady state

1.1.1	*Growth rate (g)*			
Depreciation rate (δ)	*0.02*	*0.05*	*0.10*	*0.20*
0.02	25.50	15.00	9.17	5.45
0.05	14.57	10.50	7.33	4.80
0.10	8.50	7.00	5.50	4.00
0.15	6.00	5.25	4.40	3.43
0.20	4.64	4.20	3.67	3.00
0.30	3.19	3.00	2.75	2.40
0.40	2.43	2.33	2.20	2.00

Table D Ratio of depreciation to investment in steady state

	Growth rate (g)			
Depreciation rate (δ)	*0.02*	*0.05*	*0.10*	*0.20*
0.02	0.50	0.29	0.17	0.09
0.05	0.71	0.50	0.33	0.20
0.10	0.83	0.67	0.50	0.33
0.15	0.88	0.75	0.60	0.43
0.20	0.91	0.80	0.67	0.50
0.30	0.94	0.86	0.75	0.60
0.40	0.95	0.89	0.80	0.67

Conclusion on straight-line versus geometric depreciation
Given the asset lives of plant and machinery assumed by the ONS, the equivalent geometric rate for the United Kingdom is substantially lower than its US counterpart. If the rates used to estimate UK asset stocks and depreciation were raised to US levels, the level of stocks would be lowered by a substantial amount, while depreciation as a proportion of gross investment and GDP would be simultaneously raised. These effects are quantified in Section 4 below.

Obsolescence and the interpretation of depreciation

Up to now we have treated the concept of depreciation, and the related concept of decay, as unproblematic in theory, even if difficult to measure in practice. But there are some important conceptual issues that need to be resolved. These revolve around the concept of obsolescence and have been made more acute by the rising importance of assets like computers. These depreciate rapidly but do not decay physically in any obvious sense. In this subsection we show that the basic framework is unaffected when assets suffer from obsolescence. But obsolescence does raise some tricky questions about how to measure depreciation. We show that these can in principle be resolved within an appropriately specified hedonic pricing approach.

Obsolescence versus physical decay

Some assets, like buildings, decay with age. Mechanical wear and tear causes many types of machinery to decay with use.[14] But some assets, in particular computers and software, suffer little or no physical decay but are nevertheless discarded after relatively brief service lives. The cause is usually said to be 'obsolescence', due to the appearance of newer and better models. Does this make any difference to the analysis above?

The answer is no. The weights in equation **(10)** are relative marginal products. Certainly these may decline if there is physical decay but this is not the only possibility. Anything which causes the *profitability* of capital equipment to decline will do just as well. Two possible causes of declining profitability have been identified in the literature:

1. If capital is used in fixed proportions with labour (a putty clay world), rising wages will cause older equipment to be discarded even if it is physically unchanged. As equipment ages, its profitability declines and it is discarded when profitability reaches zero. *Ex post* fixed proportions seem quite realistic for computers, where the rule is one worker, one PC. Suppose to the contrary that computer capital were malleable *ex post* and that each model is twice as powerful as its predecessor. Suppose too that the optimal capital/labour ratio is one worker to one PC of the latest type. Then the optimal ratio would be one worker to two older PCs, one worker to four PCs of the previous model, and so on. This is contrary to observation. Oulton (1995) shows that, in a putty clay world, the 'K' which should go into the production function is still one where machines are weighted by their relative marginal productivity or profitability, ie equation **(10)** still holds. Depreciation will not be geometric (since assets have a finite life) though geometric depreciation could still be a good approximation.
2. As capital ages, it may require higher and higher maintenance expenditure. This is particularly the case for computers and software, provided we understand maintenance in an extended sense to include maintenance of interoperability with newer machines and software. The profitability of a machine will then decline as it ages and it will be retired when profitability is zero. Whelan (2000a) has analysed the optimal retirement decision in such a world (although he assumes malleable capital).

Sometimes it is argued that only physical wear and tear should go into the measure of depreciation used to construct estimates of asset stocks. That part of depreciation which is due to obsolescence should be excluded. Computers (and software) do not suffer from wear and tear to any appreciable extent, so a large part of their high, measured depreciation rate must be due to

[14] Deviations from the mean rate of depreciation due to variation in the intensity of use have been estimated econometrically by Larsen *et al* (2002).

obsolescence. According to this argument, the growth rate of the US computer stock must be even faster than officially estimated by the BEA (Whelan (2000a)).

This argument is incorrect. Obsolescence, properly understood, is a valid form of depreciation. The reason is two-fold. First, as we have seen, asset mortality is part of the overall rate of depreciation. If an asset has been scrapped, then it cannot form part of the capital stock nor can it contribute to the VICS. Scrapping is of course an extreme result of obsolescence. Second, if the prices of surviving assets decline with age, then this means simply that the present value of the expected flow of services declines with age. At one level, it doesn't really matter what the cause is, the important point is that services are expected to decline. Decline could be for a whole host of reasons, including:

(a) wear and tear (which may be exogenous or may vary with use);
(b) rising costs of operating the asset; and
(c) a decrease in the *value* of the service flow even though the physical *quantity* of services is constant.

The last two possibilities are what is usually meant by obsolescence. Let us consider these in turn.

Suppose there are fixed coefficients: one person, one PC. Over time, wages are rising. Then the profitability of a PC of a given vintage will decline over time and eventually it will be scrapped when its quasi-rent falls to zero. Note that the physical capacity of the machine to produce output, and the value of that output, may be unchanging, but nevertheless the machine eventually gets scrapped because it ceases to be profitable. In this model, scrapping is endogenous: the faster the rate at which wages are rising, which depends on technical progress in the economy as a whole, the shorter the economic life of assets.(15)

The second type of obsolescence, declining value of services, could arise in the following way. Over time, new software is introduced which will not run on old machines. People prefer the new software, so the value of services from the old machines declines. Word 7 is better than Word 6 in most people's opinion, so a computer that cannot run Word 7 is worth less after Word 7 is introduced. There may be network effects here, but these do not affect the argument in the present context. You may be quite happy with Word 6 but are forced to change to Word 7 because everyone else has. But the value of the services of your old PC has still declined in the eyes of the market. And you have still voluntarily chosen to install Word 7 because you value your ability to communicate easily with other people.

Measuring depreciation in the presence of obsolescence and quality change(16)

Though this shows the correctness of incorporating obsolescence in the measure of depreciation, it is not so obvious how to do it in practice. The question is closely bound up with the issue of

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(15) This is the vintage capital model of Solow and Salter. Oulton (1995) shows that a VICS can be calculated for this model in the same way as for a more neo-classical model. However, depreciation will not be exactly geometric since assets have a finite life here.

(16) This section draws on Oliner (1993) and (1994).

adjusting prices for quality change. Both issues can be addressed in principle by employing a properly specified hedonic equation. When quality is changing we must distinguish between transactions prices and quality-adjusted prices. The transactions price of a computer is the price of a box containing a certain model computer of some specified age. It cannot be directly compared with the transactions price of a different model since the quality of the two models may differ.

Suppose we had a panel of data on transactions prices of computers of different models, covering say two years. Then we could estimate the following regression:

$$\log \tilde{p}(i,s,t) = \beta_0 + \beta_1 z(i) + \beta_2 s(i,t) + \beta_3 YD + \varepsilon(i,t), \quad t = 1,2 \qquad \textbf{(46)}$$

where:

$\tilde{p}(i,s,t)$ is the transaction price (*not* quality adjusted) of the *i*th computer that is *s* years old in year *t* (the tilde is to indicate transactions rather than quality-adjusted price)
z(*i*) is some characteristic (say speed) which affects the perceived quality of computers. In practice, there would be a vector of characteristics.
YD is a year dummy (=1 in period 2).

Suppose that we have established that the regression is satisfactory from an econometric point of view. How should we interpret the coefficients? The coefficient on the year dummy, β_3, gives the rate of growth of computer prices in year 2, with quality (*z*) and age (*s*) held constant. So this coefficient gives the rate of growth of the *quality-adjusted* price of a new computer. This is just what we need to deflate investment in current prices to constant prices in order to estimate the stock of computers in units of constant quality.

The coefficient on age (*s*), β_2, which we expect to be negative, gives the factor by which price declines with age, holding quality constant. Ie, if δ is the geometric depreciation rate, then $1-\delta = \exp[\beta_2]$: however, this is to ignore asset mortality (see below). The specification is a bit restrictive, since it constrains the rate of depreciation to be the same at all ages, the geometric assumption again. It may be that life is more complicated, but this does not change the basic principles being illustrated here.

There is another adjustment we need to make to get true depreciation. The regression suffers from survivor bias. Some assets have been thrown away and so do not get to feature in the regression. Their price can be taken to be zero (assuming that scrap value and clean-up costs cancel out). If the proportion of assets of age *s* which survive at time *t* is $\phi(s,t)$, then a proportion $(1-\phi(s,t))$ has price zero. So the price of a model of age 1 at *t*, as a proportion of the price of a new version of the same model, is not $\exp[\beta_2]$ but $\exp[\beta_2]\phi(1,t)$ and this is the true depreciation factor. If the force of mortality is geometric, then this survivor-corrected rate of depreciation is also geometric.

Note that depreciation is defined at a point in time, just as before. It is the difference between (say) the price of a new Pentium 4 and the price a one year old Pentium 4, both prices being measured in (say) January 2003. The regression will also tell us what is the difference between the price of a new 386 and a one year old 386 at the same date, even if neither price is actually observed, because the 386 is no longer manufactured. The reason is that we can measure the characteristics of the 386 and use the regression to price it.

This equation says that depreciation is a function of a computer's age, but it may rather be a function of the age of the model of which it is a particular example. That is, a new Pentium 3 and a one year old Pentium 3 might sell for the same price at a point in time (in the absence of physical wear and tear), but both computers would fall in price, and by the same proportion, when the Pentium 4 is introduced. We could test for this by redefining a computer's age in the regression equation to be the number of years since that model was first introduced. Depreciation will now not be geometric but could still be approximately so. The reason is that examples of older models will on average have higher age than examples of younger models.

Note that depreciation and scrapping will be endogenous, just as in the Salter-Solow vintage capital model. The rate of depreciation will depend on technical progress in computers and software.

Estimating depreciation in practice

Empirically, estimates of both capital stocks and services are bedevilled by two major areas of uncertainty. First, we need to know the service lives of assets. Second, there is the choice of the appropriate pattern of depreciation, and the associated pattern of decay: should we use geometric, straight-line, hyperbolic or one-hoss shay depreciation?

Service lives

Little is known about the service life of assets in the United Kingdom. Till 1983, the official estimates of the capital stock were based on the work of Redfern (1955) and Dean (1964); see also Griffin (1976). Their estimates of services lives were in turn based on the life lengths used by the Inland Revenue for tax purposes, from a period before the tax system encouraged business firms to depreciate assets more rapidly in their accounts than would be justified by true economic lives (Inland Revenue (1953)). In 1983, the Central Statistical Office (the predecessor of the ONS) revised the service lives downwards, citing (unpublished) 'discussions with manufacturers' as its authority (Central Statistical Office (1985), page 201). Following a report commissioned from the National Institute of Economic and Social Research (Mayes and Young (1994)), this reduction was reversed. But at the same time two other changes were introduced. First, a new category of asset, 'numerically-controlled machinery', was introduced into the ONS's Perpetual Inventory Model (PIM) of the capital stock. This type of asset is assumed to have a very short life by comparison with other types of plant and machinery (about 5-7 years) and the proportion of investment devoted to this type is assumed to rise over time. Second, some plant and machinery is assumed to be scrapped prematurely; the rate of scrapping is assumed to be related to the corporate insolvency rate, which has been on a rising trend since the 1970s (West and Clifton-Fearnside (1999)). Both these changes have led to a progressive shortening of the average service life of plant and machinery (but not of buildings or vehicles) in the PIM since about 1979.

Clearly then the empirical evidence for service lives in the United Kingdom is weak.[17] This judgment is confirmed by the OECD. In its capital stock manual (OECD (2001b), Appendix 3) it lists four countries (not including the United Kingdom) for which service lives 'appear to be based on information that is generally more reliable than is usually available for other countries': the United States, Canada, the Czech Republic, and the Netherlands. It is noteworthy that in each of these countries service lives are lower than assumed in the United Kingdom for both buildings and plant and machinery (at least before the effects of premature scrapping are considered).

Evidence on the pattern of depreciation

No international consensus has yet been reached on the appropriate assumption to make about depreciation (OECD (2001b)). In practice, a variety of approaches has been used. In the United States, the Bureau of Labor Statistics (BLS) produces estimates of the 'productive capital stock' or VICS that assume a hyperbolic pattern of decay rates, arguing that these are more realistic than geometric decay. The Australian Bureau of Statistics follows a similar approach (Australian Bureau of Statistics (2001)). But this pattern is not based on any strong empirical evidence.[18] Statistics Canada on the other hand uses geometric decay. The BEA does not estimate the VICS but does produce wealth measures of the capital stock using geometric depreciation (Fraumeni (1997); Herman (2000)). Jorgenson and his various collaborators in numerous studies (eg Jorgenson *et al* (1987); Jorgenson and Stiroh (2000)) have assumed geometric depreciation and decay. By contrast the ONS in common with many other national statistical agencies employs straight-line depreciation in their net stock estimates. Their gross stock estimates in effect assume one-hoss shay. No official VICS measure is published though work is currently underway within the ONS to produce one on an experimental basis.

Geometric depreciation is well supported as a rule of thumb by studies of second hand asset prices (Hulten and Wykoff (1981a) and (1981b); Oliner (1993), (1994) and (1996)). These generally find that a geometric pattern of depreciation fits the data well, even though it is frequently possible to reject the geometric hypothesis statistically. On this basis geometric depreciation has been adopted as the 'default assumption' in the US national accounts (Fraumeni (1997)). By contrast, straight-line depreciation is inconsistent with the evidence on asset prices. The stylised fact about new and second hand-asset prices is that the age-price profile is convex (OECD (2001b)). But the straight-line assumption predicts that asset prices decline linearly with age. It also predicts that efficiency declines linearly with age, before falling abruptly to zero when the asset reaches the end of its service life,[19] a pattern that may well be thought unrealistic.

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(17) Knowledge will be improved when the results of the ONS's new capital stock survey are published (West and Clifton-Fearnside (1999)).

(18) If decay is hyperbolic, the services of an asset decline at an increasing proportional rate with age. The ratio of the services from an asset which is s years old to the services from a new asset is given by the formula $(n-s)/(n-\beta s)$ where n is the service life and β is a positive parameter. One reason often cited for preferring the hyperbolic to the geometric assumption is that under geometric the largest loss of efficiency occurs in the first period of an asset's life, which is often though unrealistic. By contrast, under hyperbolic decay the losses get proportionately larger as an asset ages. However this may be, the only evidence on declining efficiency comes from studies of asset prices. There is little or no basis for estimating the additional parameter which the hyperbolic assumption requires. In practice, the BLS chooses a value of this parameter which will yield an age price profile approximately equal to that implicit in the BEA's wealth estimates, the latter of course based on the geometric assumption.

(19) This is proved in Diewert and Lawrence (2000, equation 11.22, page 281).

On these grounds, we adopt geometric depreciation for the empirical work reported below. Nevertheless considerable uncertainty still attaches to the estimated rates for different assets. In principle, the hedonic approach described in the previous subsection can be used to estimate the rates, in conjunction with data on service lives. This approach can be thought of as a rather idealised account of the method actually used by the BEA.[20] But for many assets, there is inadequate data on second-hand prices or anyway no studies have been done. Where studies have been done, they are not always up to date: much of the evidence relates to the 1970s and 1980s (Fraumeni (1997); U.S. Department of Commerce (1999)). In addition there are some methodological problems, to which we now turn.

Overestimation of depreciation when quality change is important
Suppose we estimated the regression equation **(46)** of the previous subsection with the quality variable omitted. Then the coefficient on age would pick up not only the pure age effect but also the effects of changing quality. Older models have lower quality, so the coefficient on age would be biased downwards, ie it would be more negative. It would then be wrong to estimate the stock of eg computers using quality adjusted prices, while simultaneously using depreciation rates estimated from price data which are not quality adjusted.

This is the difference between what Oliner (1993) and (1994) calls 'full' and 'partial' depreciation; see also Cummins and Violante (2002). He argues that the BEA was guilty of this error in its estimates of computer stocks. However, there has been a major revision to BEA methods since he wrote. Oliner's distinction between full and partial depreciation is referred to in a subsequent BEA methodology paper (U.S. Department of Commerce (1999)). And BEA estimates of stocks of computers and peripherals are based in part on his work.[21] The BEA's depreciation rate for PCs is now based on a more recent study (Lane (1999)). But below we argue that this study in fact suggests a lower rate than the BEA's.

What about other assets? The basis for the BEA's estimates of depreciation rates for surviving assets are the two Hulten-Wykoff studies, one for structures and the other for equipment.[22] (Estimates of asset lives, which as we have seen are also influenced by obsolescence, come from other sources.) In the structures study, the main estimates did not include quality variables. But they did try adding two quality variables to their regression: primary structural material and 'construction quality' (which they argue is 'presumably closely correlated to the availability and quality of ancillary equipment'). They also added population (derived from zip codes), which may affect land values (though land was excluded from the value of the buildings). None of these variables had much effect on the coefficient on age (their footnote 21). This suggests that quality change was not important for these assets over the period they studied.[23]

The published version of the Hulten-Wykoff equipment study does not give any details of the estimation method, so it is not clear whether their regressions included any controls for quality. But their discussion says that their estimates do not distinguish between pure ageing effects and 'obsolescence' (better referred to as quality change in our view). And Oliner refers to these

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(20) See Fraumeni (1997) for a full account.
(21) Oliner's work did not cover PCs, though the BEA used to apply his results for mainframes to PCs.
(22) See Hulten and Wykoff (1981a) and (1981b).
(23) Their data came from a survey of building owners carried out in 1972.

estimates as 'full' depreciation. So it is possible that the BEA estimates for non-computer equipment are more open to criticism on this ground and hence may overstate the depreciation rate. On the other hand, the assets actually studied by Hulten and Wykoff were machine tools, construction machinery, and autos.[24] Quality change is likely to be less important for these assets than for computers.

The case of computers

Although Oliner's work did not cover PCs, the BEA at one time applied his results for mainframes to PCs. More recently, the BEA has changed its method once again. For PCs they now rely on an unpublished study of fair market values of PCs belonging to a California-based 'large aerospace firm' (Lane 1999).[25]

For computers, two different methods were used by Lane to calculate fair market values. The first method was based on second-hand prices of computers from a variety of sources, including dealers. These were used to estimate the 'value factor' in the formula

Value = Original Cost x Value Factor

(See Lane (1999), page 12.) Note that 'Original cost' is the historic cost when the asset was new. The second method used the formula

Value = Replacement Cost New (RCN) less Normal Depreciation

where RCN is the price when new (original cost) adjusted for inflation. RCN is intended to be what an asset yielding 'comparable utility' (which we can interpret as comparable quality) would cost today. In practice this was estimated using the BLS price index for computers, which is of course adjusted for quality change. This leads to the formula:

Value Factor = RCN Factor x Percent Good

(See Lane (1999), page 17.) Here 'percent good' is the second-hand price as a proportion of the price new *at the same point in time* (not as a percent of original cost). In other words, it corresponds to the economist's concept of depreciation.

Under the first method, data were collected on the prices of a given model at various ages, which were then compared with its original cost. This information was obtained for a large number of models. Depreciation schedules showing the second-hand price as a percentage of the new price ('original cost') were plotted and a curve fitted to derive an average 'depreciation schedule' (using this term in the commercial sense, not the technical economics sense).

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(24) They also studied office equipment including (presumably) computers, but at least for computers their estimates have been superseded.

(25) In California, the property tax applies to equipment (including computers) as well as to real estate, and the base for the tax is fair market value. So there is considerable interest in valuing second-hand assets correctly. We are grateful to Richard Lane for sending us a copy of his report. The comments in the text should not be taken as critical of his study, whose focus was the correct market valuation of second-hand assets for tax purposes, not the estimation of economic depreciation in the national accounts.

The second method used a separate study of over 2000 PCs (no details given) to determine that the mean life of a PC was 34 months. The data on survival were then fitted to a theoretical survival curve (Winfrey S-0). Percent good was calculated as:

Percent Good = Annuity value of remaining service ÷ Annuity value of total service

Unfortunately, the public version of this report did not go into any detail as to how this calculation was actually done. But apparently the estimation took into account that utilisation declines over the asset's life (see Lane (1999), page 17). So percent good included some decline in service from surviving assets as well as the effect of the shorter expected life of ageing assets. The second method does seem to rest on more assumptions and on this ground the first is to be preferred.

The results are in Table E. The two methods produce similar but not identical results. Up to an age of three years, the percent good is very similar. For age above three years, percent good on the market data method is a good bit higher. At age five, percent good is 20% using market data and 6% using the survival curve approach.(26)

According to Herman (2000): 'The depreciation of PC's is now based on a California study of fair-market values of personal property including PC's [the Lane study]. The new estimates are based on a geometric pattern of depreciation that by the fifth year, results in a residual value for a PC of less than 10% of its original value. … The new method is consistent with the general procedure for calculating depreciation that was adopted in the 1996 comprehensive NIPA revision; assets are now depreciated using empirical evidence on used-asset prices and geometric patterns of price declines.' This suggests that the BEA is using the depreciation rates implied by the market data method in Lane's study.

Table E Value factors for low cost (<$50k) computers

2	*Based on market data*		*Based on survival curve*		
Age (years)	*Percent good (%)*	*Value factor (% of original cost)*	*RCN factor (%)*	*Percent good (%)*	*Value factor (% of original cost)*
	(1)	*(2)*	*(3)*	*(4)*	*(5)*
0	*100*	100	100	100	100
1	*70*	61	87	70	61
2	*47*	37	78	46	36
3	*36*	22	61	29	18
4	*30*	14	47	16	8
5	*20*	8	40	6	2
6	*15*	5	33	5	2
7	*10*	3	31	5	2

Source: Lane (1999), exhibits 5 (page 26) and 10 (page 32).
Note: Column (1), is not in Lane (1999), is calculated as 100 times column (2) divided by column (3).

(26) The results for more expensive computer systems were quite similar.

However, there is a problem. Herman's statement that the residual value of a computer is less than 10% after five years refers to column (2) ('Value factor') in the table above, not to column (1) ('Percent good'). If the geometric rate has been calculated from column (2), it would be about 40%. If from column (1), 'only' about 30%. In fact, one can calculate from the BEA's fixed asset Tables 2.1 and 2.4 that the depreciation rate on 'Computers and peripheral equipment' has been running at more than 40% since 1992 (Herman (2000)). But if (as in the present paper) we wish to define depreciation in the economic sense as the difference between the prices of a new and of a second-hand asset *at a point in time,* then we should be using the figures in column 1, not those in column 2. On this basis, the depreciation rate that the BEA is using for PCs is too high.

Conclusion on estimating depreciation in practice

1. Aggregate depreciation is the result of two forces: (a) the scrapping or retirement of assets and (b) the decline in the services yielded by a surviving asset as it ages. What is commonly called obsolescence may cause either premature scrapping or a decline in the value of services. Hence obsolescence is correctly counted as part of depreciation.
2. The hedonic pricing approach, which can be employed to measure quality adjusted prices, can also be used to measure depreciation in the presence of obsolescence. Such studies underlie the BEA's estimates of depreciation rates, including for computers.
3. While the BEA methodology seems sound in principle, the empirical evidence on depreciation is still somewhat patchy and in some cases out of date. Also it is possible that depreciation rates for non-computer equipment (insofar as they are intended to measure economic depreciation as defined in the present paper) have been overstated by the BEA, since the studies on which the BEA relies may not have controlled completely for quality change.
4. Computers (though not software) have been the subject of numerous studies which do control for quality change. The BEA has recently changed the depreciation rates used for PCs as a result of a new study. But their new depreciation rate is too high as a measure of economic depreciation: the same study from which their estimate of about 40% is derived supports a figure of about 30% as the rate of economic depreciation.

4 Capital stocks, VICS and depreciation: sources, methods and results

Sources and methods for quarterly and annual estimates of the wealth stock and VICS

The data

This section describes the sources and methods used to construct quarterly (seasonally adjusted) and annual estimates of asset stocks, the wealth measure of the aggregate capital stock, and the VICS, for the whole economy. Further details are in Appendix B.

The wealth and VICS measures are conceptually and data consistent. Both measures assume geometric depreciation and are based on the same underlying ONS series for gross investment. The only difference between them is that in weighting together the growth rates of the asset stocks, the wealth measure uses shares in the aggregate value of assets (asset price weights), while the VICS uses shares in aggregate profits (rental price weights).

Following the ONS breakdown, five types of asset are distinguished initially:

1. Other buildings and structures ('Buildings')
2. Transport equipment ('Vehicles')
3. Other machinery and equipment and cultivated assets ('Plant and machinery')[a]
4. Intangible fixed assets ('Intangibles')[b]
5. Inventories ('Stocks')

a. Includes computers and some of software investment.
b. Includes some of software investment.

Note that dwellings are excluded. The VICS and wealth measures that we present below exclude inventories. But the VICS measures are still influenced by inventories since the latter affect the estimates of the rental price weights. In effect we assume that inventories earn the same nominal rate of return after tax as do the other assets.

The official investment series include investment in computers and software but these are not distinguished separately. For some of our measures we thought it important to see the effect of greater disaggregation by asset type. We therefore developed our own estimates of computer and software investment, building on earlier work (Oulton (2001a)). These estimates derive mainly from the annual supply and use tables, supplemented by earlier input-output tables. The methods and sources are discussed below; software investment is discussed more fully in Appendix C and computer investment in Appendix D. An important issue is: given a series for real investment in computers, and a series for total real investment in plant and machinery, ie including computers, how should one derive an index for real investment in plant and machinery *excluding* computers? This is discussed in Appendix D; note that simply subtracting computer investment from the total is the wrong answer.

Method

Quarterly estimates

When rental prices were calculated using the quarterly model, the estimates of the rental price weights were excessively volatile. So we adopted a two stage procedure: (1) estimate the rental price weights using *annual* data; (2) estimate the quarterly VICS using *quarterly* capital stock data and *annual* rental price weights. Even on annual data, the rental prices needed some smoothing (see below).

To estimate the quarterly VICS, the *quarterly* growth rates of asset stocks are weighted together using the *annual* rental price weights. But here we revert to the simpler model of Oulton (2001b) and now assume that capital services in period t are proportional to asset stocks in place at the end of t-1:

$$K_{it} = A_{i,t-1}, \qquad i = 1,...,m \qquad \textbf{(47)}$$

We also measure the growth rate of the wealth measure on an end-of-period basis:

$$\ln\left[A_t / A_{t-1}\right] = \sum_{i=1}^{m} \bar{v}_{it} \ln\left[A_{it} / A_{i,t-1}\right],$$
$$\bar{v}_{it} = (v_{it} + v_{i,t-1})/2, \; v_{it} = \frac{p_{it}^A A_{it}}{\sum_{i=1}^{m} p_{it}^A A_{it}}, \quad i = 1,\ldots,m \qquad \textbf{(48)}$$

Consequently, when comparing the quarterly (but not the annual) VICS and wealth measures, *the latter should be lagged one quarter*.

Cyclical scrapping
So far we have assumed that the depreciation rate for each asset type does not vary over time. Each rate includes an allowance for scrapping at some 'normal' rate. But arguably assets are more likely to be scrapped in a recession and certainly this is consistent with much anecdotal evidence.[27]

One might argue that the life of capital assets is prolonged during a boom: assets are not scrapped when they normally would be but are retained in order to meet high demand. However obsolescence may be more rapid during a boom, ie lives are shorter, since firms are happy to buy the latest kit; this may be particularly the case for high-tech assets. On the other hand during a recession firms who wish to maintain their capital at its current level may be more cautious about replacing it at the normal rate, due to financial constraints and higher perceived risk, so lives get longer. We have no evidence on the relative strength of these opposing tendencies.

We assume that the mechanism is asymmetric. Plant and machinery (including computers and software) is assumed to be scrapped at an accelerated rate when output falls, but there is no corresponding mechanism when output rises. The effect is only temporary: the assets which get scrapped would have been scrapped in the end anyway. So this mechanism makes the time path of the plant and machinery stock more cyclical. Buildings and vehicles are assumed not to be subject to cyclical scrapping. Buildings may stand idle but in a recession they are not assumed to be physically destroyed more rapidly than normal. One justification is that buildings, like vehicles, usually have broad second-hand markets, hence scrapping would not generally be profitable. Plant and machinery on the contrary is frequently highly specialised and second-hand markets are thin. Also, if a machine can be sold, it may be for export abroad, in which case it ceases to be part of the UK capital stock.

If the fall in output is expected to be only temporary, then it would be irrational to scrap capital. But if the fall is expected to be *permanent*, it is reasonable that firms would adjust their capital stocks proportionately. We interpret a fall as permanent when it is a fall when measured on an annual basis. From 1973 to 2001, there were nine years in which manufacturing output [ONS code CKYY] fell. The largest fall was in 1980, 9.1%. But one can calculate that non-manufacturing output has never fallen on an annual basis over this period. Hence the cyclical scrapping mechanism is assumed to apply only to manufacturing. Let p be the proportion of the plant and machinery stock which is located in manufacturing: between 1973 and 1999, this proportion fell from 43% to 30%. Then in years when output falls in manufacturing, our model

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[27] This issue attracted attention following the recession of 1979-81 which was particularly deep in manufacturing. The widely varying estimates of premature scrapping in that period are reviewed in Oulton and O'Mahony (1994, chapter 3).

assumes that the whole-economy plant and machinery stock (including computers and software) is reduced by p times the fall in manufacturing output.

The sensitivity of our estimates to this cyclical scrapping assumption is explored further below.

Estimates of capital stocks and VICS

In this subsection we present our estimates of wealth and VICS constructed under a range of assumptions about depreciation and asset lives. The estimates of growth rates are calculated using equations **(36)** and **(37)** in Section 2. The estimates of levels in constant prices are calculated by assuming that the nominal and real values are the same in 1995 Q2 and then applying the growth rates to these values. 'Whole-economy' real growth rates of wealth and VICS are chain-weighted aggregates of asset level growth rates, ie they are Törnqvist indices. The weights are the nominal wealth shares in the wealth measure and the nominal profit shares in the VICS measure. All our measures assume that depreciation is geometric.

The variants

We have constructed six variants of each of our wealth and VICS measures. Since, as we have seen, there is considerable uncertainty about the true length of asset lives, our assumptions are designed to illustrate the extent of the corresponding uncertainty about the level and growth rate of capital. A second aim is to show the effect of a more detailed disaggregation by asset type. The variants are described in the table overleaf.

These variants share a common dataset — the underlying investment and profits series are identical inputs into both. Unless stated otherwise, all data (including investment price deflators) are consistent with the UK national accounts. The dataset is described more fully in Appendix B. The UK national accounts provide quarterly constant and current price investment data at a five-asset level: dwellings, buildings, other machinery and equipment (ie, plant and machinery), transport equipment (ie, vehicles) and intangible fixed assets. Dwellings are excluded from all our calculations.

The choice between wealth and VICS measures depends on the purpose at hand. The six variants for each measure are designed to throw light on the quantitative importance of methodological and data uncertainties. To see the effect of different asset life assumptions, variants BEA, ONS1 and ONS2 were constructed. Variant BEA is constructed using the four principle assets and uses the BEA's asset lives and corresponding (geometric) depreciation rates. Variant ONS1 is also constructed using the four principle assets but employs the official (ONS) asset lives combined with US estimates of the declining balance rate in the formula for the (geometric) depreciation rate.[28]

[28] The formula for the depreciation rate is R/L where R is the 'declining balance rate' and L is the asset life (see page 27). See Table B.2, Appendix B for the depreciation rate calculations for ONS1.

	Variant	*Description*[29]
1	BEA	Aggregated from four assets: buildings, plant, vehicles, intangibles Depreciation Rates: consistent with US NIPA ('BEA consistent')
2	ONS1	Aggregated from four assets: buildings, plant, vehicles, intangibles Depreciation Rates: calculated using asset lives consistent with UK national accounts ('ONS consistent')
3	ONS2	Aggregated from four assets: buildings, plant, vehicles, intangibles Depreciation Rates: geometric rates equivalent to ONS straight-line rates
4	ICT1	Aggregated from five assets: buildings, plant, vehicles, intangibles, computers Depreciation Rates: consistent with US NIPA ('BEA consistent') Computer price index: consistent with UK national accounts
5	ICT2	Aggregated from five assets: buildings, plant, vehicles, intangibles, computers Depreciation Rates: consistent with US NIPA ('BEA consistent') Computer price index: US computer price index, adjusted for exchange rate changes
6	ICT3	Aggregated from six assets: buildings, plant, vehicles, intangibles, computers, software Depreciation Rates: consistent with US NIPA ('BEA consistent') Computer price index: US computer price index, adjusted for exchange rate changes Software price index: US price index for pre-packaged software, adjusted for exchange rate changes

The wealth measure of ONS1 comes closest in spirit to the official, net stock measure of the UK capital stock. However, there are three differences. First, in the ONS model, the service life of particular asset can vary across industries, whereas we use the same life in all industries. Second, to aggregate capital stock across asset types, the ONS simply sums the stocks. We on the other hand use chain-linking (as will the ONS from 2003). Thirdly, the ONS assumes straight-line depreciation whereas we assume geometric.

Variant ONS2 is identical to variant ONS1 except that the depreciation rates for buildings and plant are calculated as 'steady-state values'. Section 3 (Table B) provided a motivation for the use of these steady-state values.

ONS series for investment do not break out investment in computers or software separately. Computers are subsumed in the plant category and software is split between plant and intangibles. To see if asset composition has an important effect on the aggregate measures, variants ICT1, ICT2 and ICT3 were constructed.

[29] In variants ICT1-ICT3 plant is actually 'rest of plant' and in variant ICT3, intangibles is actually 'rest of intangibles'. The reason for this is explained on the next page.

Variant ICT1 treats computers as a separate asset, uses BEA consistent asset lives and uses the UK investment price deflator for computers.(30)(31) The plant category is now just the 'rest of plant' to avoid double counting.

Variant ICT2 is identical to variant ICT1 in all aspects except that it uses the official US (BEA) price index for computers, adjusted for changes in the sterling dollar exchange rate. As mentioned in Section 3, this index is constructed using hedonic techniques.

Variant ICT3 builds on variant ICT2 by breaking out software as a separate asset (in addition to computers). We use a BEA research series for the software investment deflator, the price index for pre-packaged software, (32) and apply the 'times 3 adjustment' as described in Oulton (2001a). The plant category is now the 'rest of plant', ie computers and software are excluded, and the intangibles category is now the 'rest of intangibles' after excluding the part of software previously included here. Obtaining software series for investment and then apportioning it to plant and intangibles investment is described in Appendix C. The methodology adopted to calculate the 'rest of plant' and the 'rest of intangibles' in constant prices is described in Appendix D.(33)

Table F: Depreciation rates for each variant by asset (per cent per annum)

Variant	Buildings	Plant and Machinery	Vehicles	Intangibles	Computers	Software	*Price index for computers /software*
BEA	2.50	13.0	25.00	22.0	--	--	--
ONS1	1.14	5.69	20.59	22.0	--	--	--
ONS2	2.03	7.57	20.59	22.0	--	--	--
ICT1	2.50	13.0	25.00	22.0	31.50	--	UK
ICT2	2.50	13.0	25.00	22.0	31.50	--	US
ICT3	2.50	13.0	25.00	13.0	31.50	31.50	US

The annual geometric depreciation rates used in the variants are shown in Table F. (34) The depreciation rate on intangibles is kept at the BEA rate of 22% per annum for all the variants except variant ICT3 which treats software as a separate asset. In the UK national accounts, part

(30) ONS code: PQEK.
(31) With Blue Book 2003, the aggregate ONS capital stock series will consider computers separately as an asset (giving them a shorter life length of five years) but the investment deflator for computers will be the same as that for plant and machinery as a whole.
(32) Parker and Grimm (2000).
(33) In nominal terms, separating out computers and software from plant and software from intangibles to get 'rest of plant' and 'rest of intangibles' is easy: simply subtract the investment in these subcategories from the total. However, to get constant price series additional calculations are needed because for their constant price estimates the ONS changes the weights about every five years.
(34) The analysis always refers to the annual depreciation rates. However, these annual rates are converted to quarterly rates for the calculations since the data are on a quarterly frequency.

of software is in intangibles and part in plant so separating software from the intangibles basket will correspondingly lower the depreciation rate on the rest of intangibles.[35]

The results: an overview

Estimates of levels and growth rates are presented for the period 1979 Q1-2002 Q2, though the calculations were actually done from an earlier date.[36] Average growth rates of the aggregate wealth and VICS measures are given in Table G and standard deviations of these growth rates are given in Table H. Average shares[37] in wealth and profits are shown in Table I and the growth rates of the individual assets are given in Table J. Graphs of the shares over the entire sample period are presented in Appendix E.

Considering first the wealth measure, we find little difference across variants in the average growth rate over the whole period 1979 Q1-2002 Q2 (Table G). The growth rates do not appear much affected by differences in asset composition or asset lives. This is because buildings have by far the largest share in the wealth measure but show little difference in their average growth rate across variants. Whatever variation there is appears to be caused by changes in the growth of plant stock (Graphs E.1 and E.2 in Appendix E plot the growth rates of buildings and plant for variants BEA and ICT3 as an example).

The VICS measure on the other hand shows larger differences between variants. This is because the VICS measures give greater weight to assets whose rental prices are high in relation to their asset prices and which are growing rapidly. Thus it is not surprising that VICS variant ICT3 has the highest average growth rate. Over 1990 Q1-2002 Q2, its quarterly growth rate was some 0.4 percentage points faster than that of wealth variant ICT3. Variant ICT3 treats computers and software separately. These assets have short service lives and falling asset prices, consequently their rental price is high relative to their asset price. This combined with rapid growth in these stocks (Table J) means that the VICS, which weights by rental price, grows more quickly than the wealth measure. This story is particularly true of the 1990s, where computers and other high-tech assets have become increasingly important.

VICS variant BEA, while having a slightly higher average growth rate in the 1990s than the corresponding wealth measure, does not show the complete picture. This is because it does not recognise that the asset composition mix in the 1990s had shifted to fast growing assets with shorter lives. The combined rental weight of ICT in variant ICT3 over the entire period is 8% on average (Table I). This multiplied by high growth in these assets (Table J) gives VICS variant ICT3 an added boost.

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(35) Variants ICT1-ICT3 treat computers as a separate asset. As mentioned earlier, computers are included in plant and machinery investment by the ONS, so separating them out from plant should arguably lower the depreciation rate for the remainder of plant. However, the depreciation rate used in the United States by the BEA for plant and machinery excluding computers and software is about 13% for the entire time period (including the 1990s), so we use this rate for the rest of plant in Variants ICT1-ICT3.

(36) The calculations are actually done for from 1965 Q1 onwards (with starting values for the asset level stocks set in 1962 Q1) for variants BEA, ONS1 and ONS2. Similarly, calculations for ICT1-ICT3 are done from 1976 Q1 onwards. Estimates are presented from 1979 Q1 onwards because by this time the impact of the initial values of the stocks (which derive from historic data of lower quality) is minor, or even negligible in the case of the short-lived assets. The wealth and VICS estimates under a variety of assumptions can be downloaded from the Bank of England's website (www.bankofengland.co.uk/workingpapers/capdata.xls).

(37) Shares may not add up to 100 due to rounding.

The most striking contrast in Table G is between wealth measures which implicitly use UK price indices to deflate ICT assets (BEA, ONS1, ONS2, and ICT1) and VICS measures which employ US methods (ICT2 and ICT3). Thus we find that in the 1990s the quarterly growth rate of the ICT3 VICS measure was 0.3 percentage points higher than that of the BEA wealth variant.

Table G: Average growth rates (per cent per quarter, chain-linked)

Variant	*1979 Q1-2002 Q2*		*1979 Q1-1989 Q4*		*1990 Q1-2002 Q2*	
	Wealth	**VICS**	**Wealth**	**VICS**	**Wealth**	**VICS**
BEA	0.76	0.81	0.71	0.69	0.80	0.91
ONS1	0.79	0.81	0.74	0.69	0.83	0.91
ONS2	0.73	0.76	0.65	0.62	0.79	0.88
ICT1	0.67	0.78	0.68	0.71	0.66	0.84
ICT2	0.68	0.87	0.70	0.82	0.67	0.92
ICT3	0.72	1.04	0.74	0.98	0.71	1.10

Table H: Standard deviation of growth rates (per cent per quarter)

Variant	*1979 Q1-2002 Q2*		*1979 Q1-1989 Q4*		*1990 Q1-2002 Q2*	
	Wealth	**VICS**	**Wealth**	**VICS**	**Wealth**	**VICS**
BEA	0.22	0.38	0.26	0.43	0.15	0.30
ONS1	0.17	0.31	0.21	0.34	0.12	0.24
ONS2	0.20	0.34	0.24	0.37	0.13	0.26
ICT1	0.21	0.34	0.25	0.40	0.17	0.25
ICT2	0.21	0.38	0.25	0.43	0.18	0.32
ICT3	0.23	0.45	0.26	0.50	0.19	0.39

Table I: Average shares in nominal wealth (W) and profits (P) by asset: 1979 Q1-2002 Q2

(per cent)

Variant	**Buildings**		**Plant**		**Vehicles**		**Intangibles**		**Computers**		**Software**	
	W	**P**	**W**	**P**	**W**	**P**	**W**	**P**	**W**	**P**	**W**	**P**
BEA	71	47	24	39	4	11	1	3	--		--	
ONS1	65	44	30	43	4	11	1	3	--		--	
ICT1	70	48	23	34	4	11	1	3	1	4	--	
ICT2	70	48	23	34	4	11	1	3	1	4	--	
ICT3	70	46	23	33	4	11	1	2	1	4	1	4

Table J: Average growth rates of asset stocks: 1979 Q1-2002 Q2 (per cent per quarter)

Variant	**Buildings**	**Plant**	**Vehicles**	**Intangibles**	**Computers**	**Software**
BEA	0.70	1.04	0.18	0.94	--	--
ONS1	0.73	0.98	0.19	0.94	--	--
ICT1	0.70	0.47	0.18	0.94	5.21	--
ICT2	0.70	0.47	0.18	0.94	7.25	--
ICT3	0.70	0.45	0.18	0.91	7.25	5.89

The VICS measures have higher average growth rates than their wealth counterparts and their growth is also more volatile, when volatility is measured by the standard deviation (Table H). This higher volatility does not have a simple explanation. Partly it is due to the more volatile assets receiving larger weight in the VICS, partly to the fact that profit shares are more dispersed than wealth shares,[(38)] and partly to an interaction between shares and growth rates.

Across variants, shares are also influenced by differences in assumed depreciation rates, since the latter cause changes in the level of the asset stocks which influence the calculation of the shares. Thus in Table I when the depreciation rate on plant and buildings changes between the BEA and ONS1 variants, the wealth and profit shares also change. Separating out computers changes the profit share of plant more than its wealth share (BEA versus ICT1). Two forces are at work here. First, a change in the depreciation rate for 'rest of plant' causes some change in the 'rest of plant' stock which affects both measures. But second, the depreciation rate has an additional impact through the formula for the rental price, equation **(34).**

As we will see in more detail below, constant price levels of wealth do not differ substantially across variants when the asset composition or the deflators change (Chart 9). However, in the discussion of straight-line versus geometric depreciation in Section 3 it was pointed out that changes in the asset level depreciation rates will affect the levels of the aggregate stock. This shows up clearly when we compare the levels for variants BEA, ONS1 and ONS2 (Chart 5).

In the remainder of this subsection we look at the sensitivity of the estimates in more detail. Specifically, we consider their sensitivity to (a) variations in asset life; (b) separating out computers and software; (c) the choice of price index for computers; (d) the method of aggregation, chain-linking or fixed-base; and (e) cyclical scrapping.

Sensitivity of estimates to asset life assumptions
In Section 3 we discussed straight-line as an alternative to geometric depreciation. In the US NIPA, depreciation is assumed to be (in most cases) geometric (Fraumeni (1997)), while the ONS (along with many other national statistical agencies) assumes straight-line depreciation. A related issue is the service life assumed for each asset. In general, the service lives assumed by the ONS are longer than in many other countries, as we have seen.

Variant BEA uses US geometric depreciation rates whereas ONS1, while geometric, uses UK asset lives. (See Table B.2 in Appendix B for the calculations.) Charts 1 and 2 show that using longer, UK asset lives does not have a significant impact on the growth rate of wealth and VICS.

As pointed out in Section 3, quantitative comparisons between straight-line and geometric depreciation are not easy because in the straight-line case, the depreciation rate depends on the age structure of the stock. But if we assume that investment in an asset is growing at a constant rate over the assumed life of the asset, then we can calculate a steady-state depreciation rate in the straight-line case that can be used to compare to the geometric case.[(39)] The longer UK asset lives

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(38) See Charts E.5 -E.16 in Appendix E for a graphical representation of this.
(39) Clearly the comparison is trivial if the steady-state depreciation rate associated with the straight line case is the same as the geometric rate.

coupled with the straight-line assumption means that the steady state depreciation rates corresponding to these asset lives are smaller than the BEA consistent geometric rates. (See Table B, Section 3.)

Chart 1

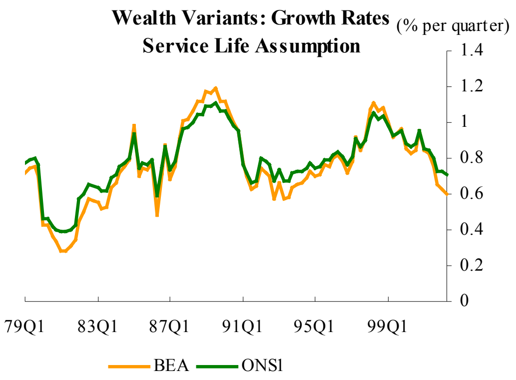


Chart 2

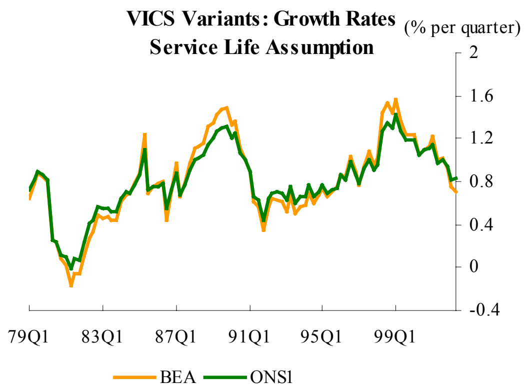


Chart 3

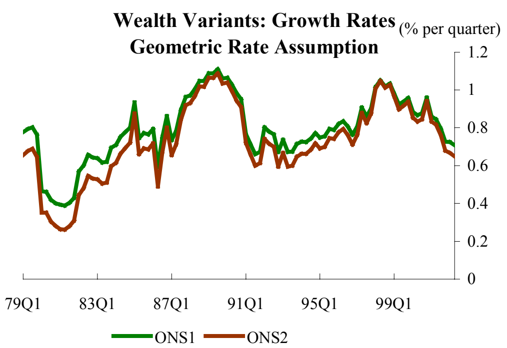
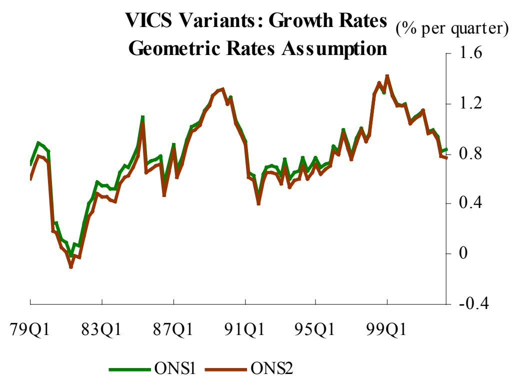


Chart 4



Variant ONS2 uses steady-state values for buildings and plant instead of the declining balance values used in ONS1.[40] Charts 3 and 4 compare wealth and VICS growth rates for these variants. They show that even large differences in asset life assumptions (eg, for plant) do not appear to make much of a difference to the aggregate growth rates, at least towards the end of the sample period.

However, different depreciation rates have a substantial effect on the *levels* of the stocks, as was noted in the discussion of straight-line versus geometric depreciation in Section 3. If the rates used to estimate UK asset stocks were the higher US ones, the level of the stocks would be lowered by a large amount. This is evident from Chart 5 when comparing BEA to ONS1/ONS2: higher depreciation rates shift the whole profile of wealth downwards.

[40] Since vehicles and intangibles already had relatively high depreciation rates in ONS1, they were kept unchanged in ONS2.

Chart 5

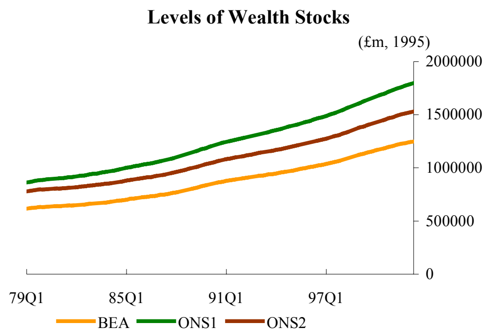
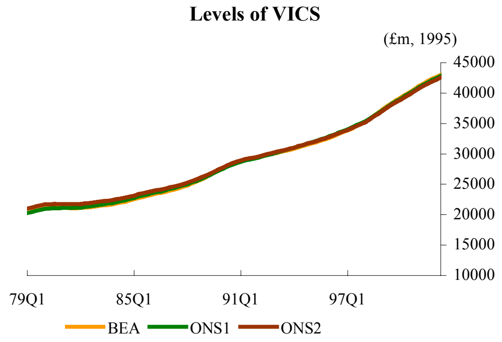


Chart 6



The effect on the level of the VICS is necessarily smaller (Chart 6). This is because in all variants the real level of the VICS in the base period (1995 Q2) is equal to the nominal level (profit in current prices) in that period. So different assumptions about depreciation simply rotate the VICS profile about the fixed point in 1995. In other words, before and after the base year the estimated level will be affected by different assumptions about depreciation only to the extent that the growth rates are affected. By contrast, though for all variants nominal and real investment levels are equal in 1995 Q2, there is no comparable constraint on the real and nominal level of wealth: these can and do differ in the base period.

Asset composition: separating out computers and software

We start by comparing BEA with ICT1 in Charts 7 (wealth) and 8 (VICS). The depreciation rates on all 'traditional' assets are the same, but in ICT1 computers have been separated out from the plant and machinery category and now are depreciating at a higher rate.(41) Separating out computers from plant has the apparently odd effect of lowering the growth rate of the aggregate wealth measure. To understand this, recall that computers are subsumed in the plant category for variant BEA (and for ONS1 and ONS2) and software is subsumed in intangibles and plant in all variants except ICT3. The average quarterly growth rate of plant (including computers) is around 1% (BEA, ONS1 and ONS2). When we separate computers (and/or software), the average quarterly growth of 'rest of plant' falls to about 0.5% (Table J). In fact, since 2001, the growth rate of 'rest of plant' is negative. The impact of the rapid growth of computers is counterbalanced by the small share of computers in wealth (1%), so the overall effect is to drag down the growth rate of the wealth measure when computers and/or software is included. The impact of this drag stemming from the 'rest of plant' is less severe in the VICS measures because of the growing share of computers and software in profit; the latter interacts with the very rapid growth of ICT asset stocks.

On the VICS measure, variant ICT1 (as compared to variant BEA) is also affected by the drop in the growth rate of the 'rest of plant' asset. But this is offset to a large extent by computers which are not only growing fast but also have a growing share in nominal profits.

(41) In Variant BEA they had a depreciation rate equal to that of plant (13%). In Variant ICT1, the rate is 31.5%.

Chart 7

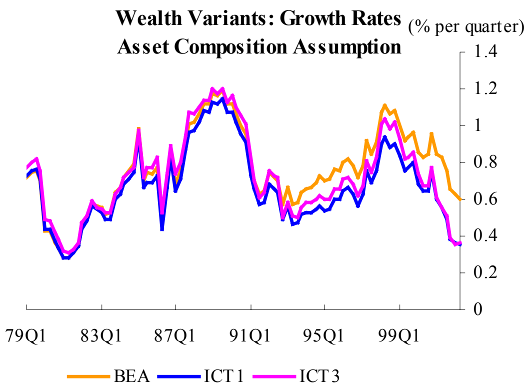
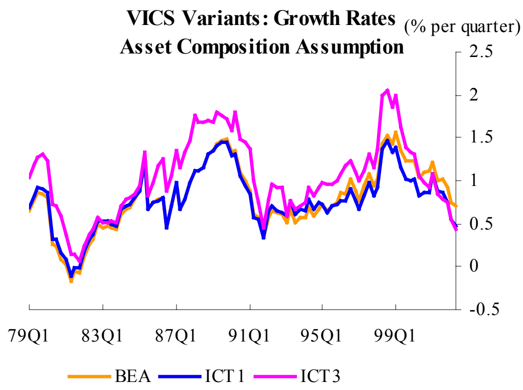


Chart 8



Software is another high depreciating, fast growing asset with a high rental to asset price ratio. While a strict comparison between ICT1 and ICT3 cannot be made,(42) it is interesting to see the impact of treating software separately. Growth rates of both wealth and VICS are higher though VICS shows the larger increase, especially in the mid to late 1990s. In level (real wealth) terms, there is not much of a difference (Chart 9). The higher BEA growth rates in the 1990s let the BEA estimate of the wealth level catch up with the ICT3 estimate. A similar picture emerges for the corresponding VICS levels (Chart 10).

Chart 9

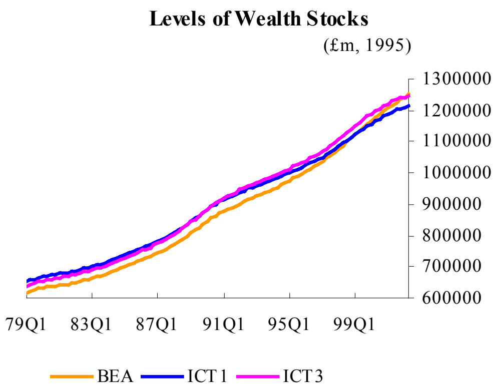
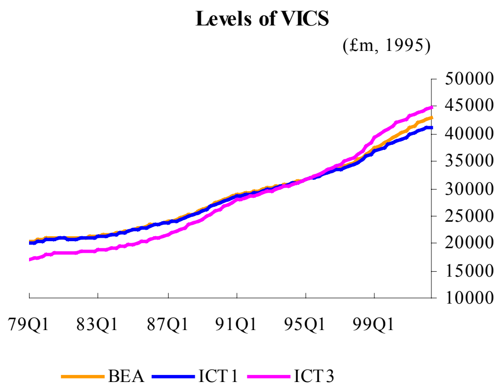


Chart 10



Now compare variants ICT2 and ICT3. They use the same (US, hedonic) price deflator for computers but variant ICT3 separates out software as an asset. As expected, the growth rates of the wealth measures show little difference (Chart 11) because even though software is growing fast, it has a very small wealth share. On the VICS measure, variant ICT3 shows strong growth in the mid to late 1990s (Chart 12), as result of rapid growth in the software stock and because the share of software in profits is higher than in wealth.

(42) ICT3 uses a different investment price deflator for computers.

Chart 11

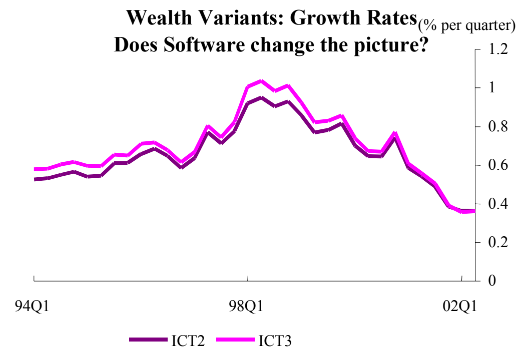
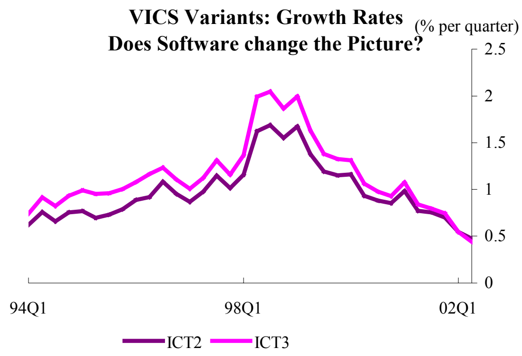


Chart 12



Sensitivity of the estimates to the price index for computers

The sensitivity of the wealth and VICS estimates to changes in the investment price deflator for computers follows a familiar pattern. Variant ICT1 uses the official UK national accounts producer price index for computers whereas variants ICT2 (and ICT3) use the US (hedonic) price deflator. The US computer investment price deflator has fallen faster than the UK one – hence when growth in the volume measure is considered, the asset stock is growing faster in variant ICT2 than ICT1 (see Table J). This larger growth, *ceteris paribus*, increases the growth of the aggregate measure. The impact is however muted in the wealth measure because the share of computers in wealth is very small (Chart 13). But as Chart 14 shows, the effect on VICS variant ICT2's growth rate is larger because of the share of computers in profit is greater than in wealth. The difference in wealth levels is negligible.

Chart 13

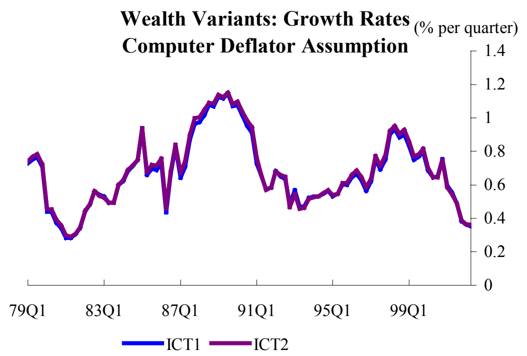
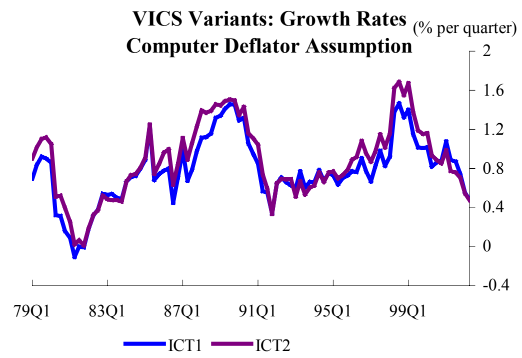


Chart 14



Sensitivity of the estimates to the method of aggregation: chained or fixed weights

The estimates presented so far are all quarterly chain-linked. It is generally agreed that fixed weight indices can be highly misleading when relative prices are changing rapidly. Current ONS methodology[43] is to change the weights every five years or so for most variables.[44] At the moment, the national accounts use 1995 weights for measuring growth rates from 1994 onwards.

(43) See ONS (1998), page 221.

(44) The ONS capital stock is a simple sum of asset level stocks. With the ONS shifting to annual chain-linking in 2003, the official capital stock series will also be chain-linked then.

Over this period the relative price of computers has been falling rapidly, even employing the UK price index. To see the effect of chain-linking, we calculated 'fixed but periodically updated' indices of wealth and VICS (following ONS methodology), for all variants. Comparisons of the ONS1, BEA and ICT3 variants are presented in Charts 15-20.

For variants ONS1 and BEA, chained and fixed-weight aggregates are similar (Charts 15a, 15b, 16a, 16b). This is because the assets that experienced large relative price changes (computers, software, telecommunications) are subsumed in the larger asset categories and relative price changes between buildings on the one hand and plant and machinery or vehicles on the other have not been large.

Chart 15a

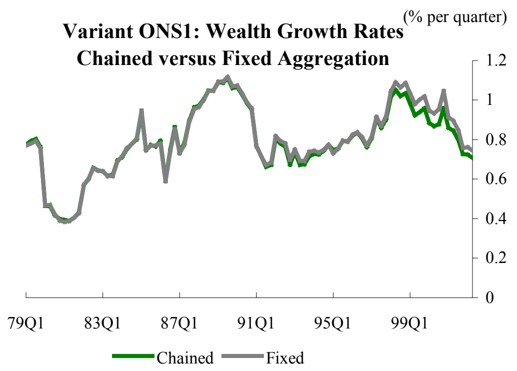


Chart 15b

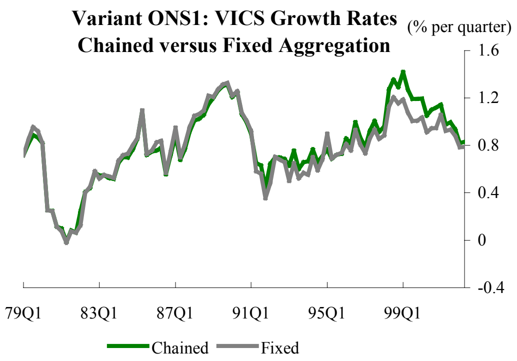


Chart 16a

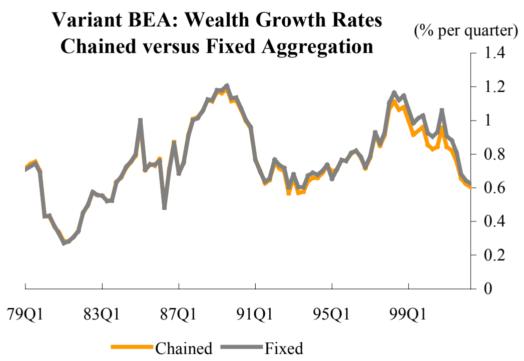
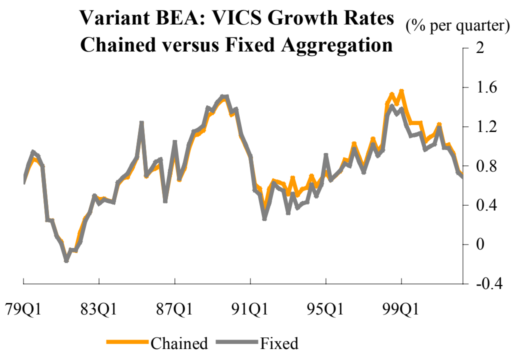


Chart 16b



The same cannot be said for variant ICT3 (Charts 17,18). Both the wealth and VICS growth measures show that the chained growth rates are lower for the most part than the fixed-base ones, as we would expect. The latest fixed base in use by the ONS is 1995. Since 1995, the investment price of computers and software has fallen rapidly. The fixed-base aggregate uses a share calculated on the higher 1995 prices (relative to the period that followed).

Chart 17

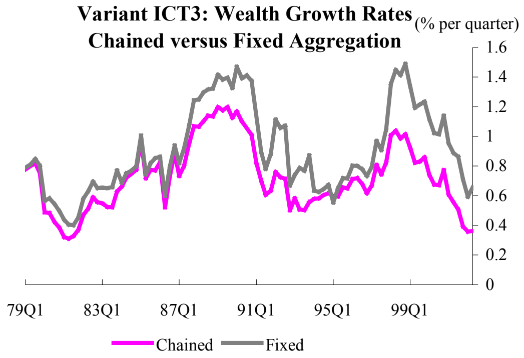
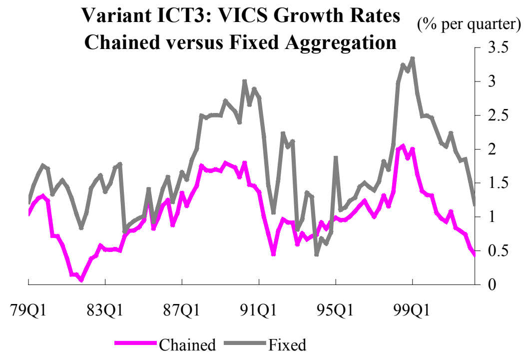


Chart 18



A similar consistent picture emerges in the levels context. Reflecting the high level of aggregation, chained and fixed-base levels of the real wealth measures of variants ONS1 and BEA show little difference (Chart 19a, 19b). Large price movements in certain assets create a divergence in levels of chained and fixed-base measures of variant ICT3 (Chart 20).

Chart 19a

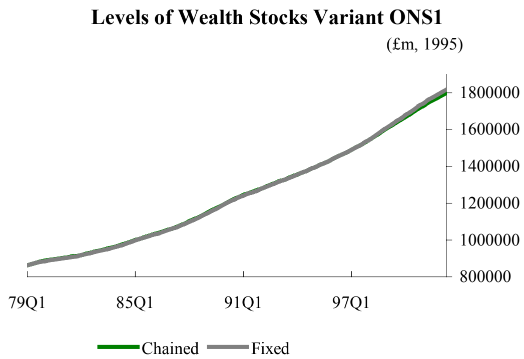


Chart 19b

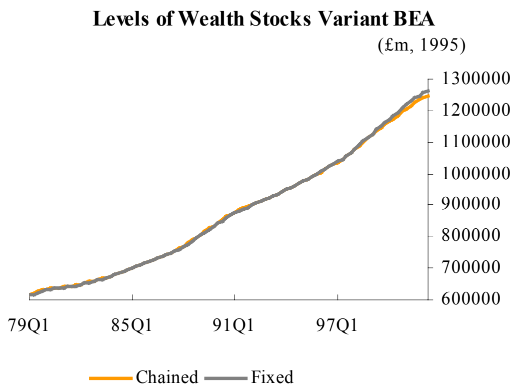
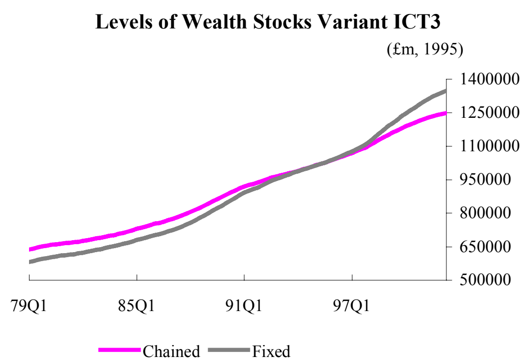


Chart 20



The cyclical scrapping assumption

All variants have embedded in them a cyclical scrapping mechanism, described more fully earlier in this section. This affects only plant and machinery, computers and software, but no other asset types. The mechanism is asymmetric. Plant and machinery (including computers and software) is scrapped when output falls, but there is no corresponding effect when output rises. Since there have been no occasions during our sample period when non-manufacturing output has suffered an absolute fall on an annual basis, cyclical scrapping affects only manufacturing. This mechanism makes the time path of the capital stock more cyclical.

Chart 21

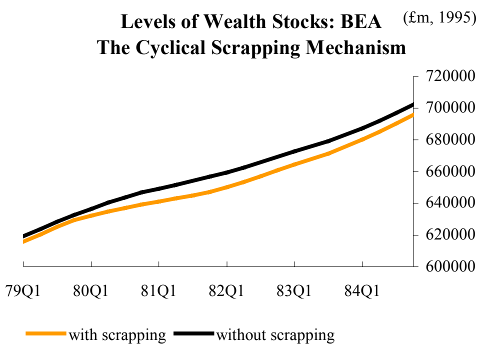


Chart 22

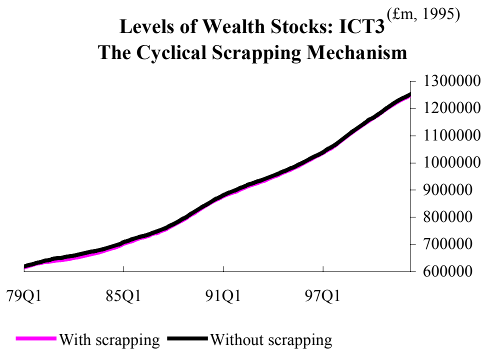


Chart 23a

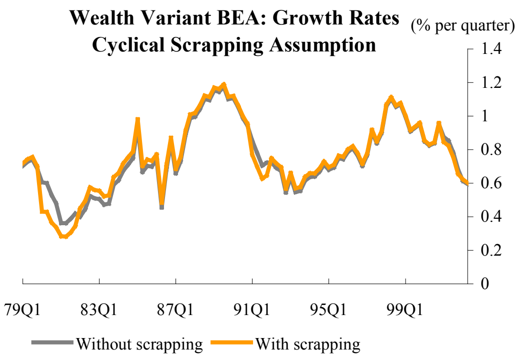


Chart 23b

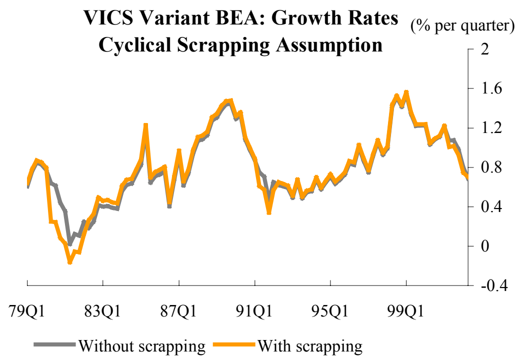


Chart 24a

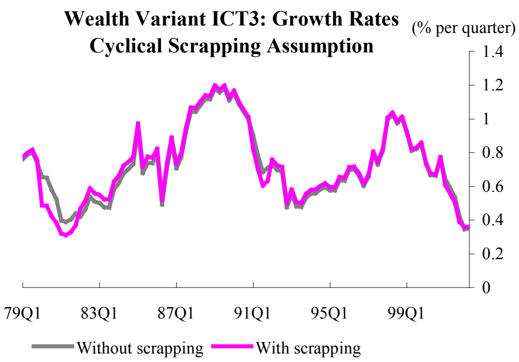
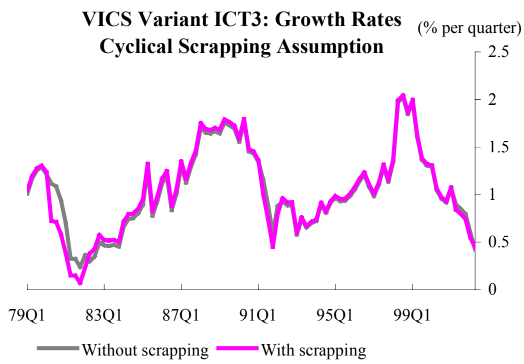


Chart 24b



One could argue that the cyclical scrapping of plant could show up as a shorter life length (and thus higher depreciation rate) in the data in years when it occurs. In other words, there is no need to adjust the stock of plant in manufacturing for cyclical scrapping because the depreciation rate applied to plant in those years already embodies the effect. This could be the case if the depreciation rate used for plant was time varying. However, we use a constant depreciation rate which we assume is the rate in 'normal' years. Hence we add on the mechanism described earlier in this section for certain years to adjust the depreciation rate for cyclical scrapping.

The impact on the level of the wealth stock is minor, since by construction the mechanism is temporary (Charts 21 and 22). There is a similar pattern to the level of the VICS measures. The growth rates of wealth (Charts 23a and 24a) and VICS (Charts 23b and 24b) become more pronounced in downturns (the recession years 1979 and 1991) but catch up in upturns. So there is only a small effect on the overall level by the end of the period.

Estimates of aggregate depreciation

Aggregate depreciation (capital consumption) is the difference between gross and net domestic product. In the past, net domestic product (NDP) has received much less analytical attention than gross. But Weitzman (1976) argued that net, not gross, domestic product is the appropriate measure of welfare.[45] And King (2001) has argued that GDP may be a misleading measure of output when the mix of assets is shifting towards shorter-lived ones, a situation where aggregate depreciation may be rising. This subsection presents estimates of the aggregate nominal depreciation rate and depreciation (as a percentage of GDP) for the six variants. We use the nominal measure because it has a natural interpretation. Section 3 showed that the aggregate nominal depreciation rate can be thought of as a weighted average of the depreciation rates on individual assets, where the weights are the shares of each asset in aggregate nominal wealth.

Variants BEA and ONS1 were constructed on the basis of different depreciation rates at the asset level (for plant and buildings). Variant BEA has the higher US rates while ONS1 has lower rates, corresponding to the longer service lives assumed by the ONS. The discussion of straight-line depreciation in conclusion to Section 3, noted that, if the depreciation rates used to estimate the UK stocks were raised to US levels, then the aggregate depreciation rate would be raised. This is evident from Chart 25. The nominal aggregate depreciation rate on the BEA variant is almost twice that of the ONS1 variant. However, while there is a large difference in the levels of the aggregate rate, the two variants show a similar pattern of variation over time.

Chart 26 shows a comparison of nominal depreciation rates between the United Kingdom and the United States.[46] The US rate has risen fairly steadily over two decades, standing now at about 9% per annum. Unlike in the United States, and despite its deliberate methodological similarity, in the UK variant BEA does not show any tendency to increase substantially in the 1990s (nor does variant ONS1 as is clear from Chart 25). The reason is that buildings (which have the largest share in nominal wealth) continued to dominate in this period. Plant grew faster than buildings (compare Charts E.1 and E.2 in Appendix E) but its price (relative to buildings) fell so that its share in the total did not increase. Variant ICT3 is the closest methodologically to the US NIPA. It shows a slightly larger rise in the 1990s than does variant BEA (see also Chart 27) but

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(45) In Weitzman's concept, NDP is deflated by the price index of consumption. Weitzman's concept is discussed further in Oulton (2002).

(46) The UK coverage is whole economy less dwellings and the US coverage is the private non-residential sector.

still does not match the upward trend in the United States. This is because the faster growing assets experienced large price falls and their shares in nominal wealth were very small. To replicate the US nominal depreciation rate experience, the shares of computers and software in the UK wealth stocks would need to be higher than they are at present.

Chart 28 presents depreciation as a proportion of GDP (in current prices) for all the measures.(47) It also includes the ratio calculated from ONS data (labelled 'official'). The 'official' series appears to have a downward trend.

It is noteworthy that even though the nominal aggregate depreciation rate shows an upward trend in the United States, depreciation as a proportion of GDP is almost flat (Chart 29) up to around 1999. But both variant ICT3 for the UK and the US ratio show an uptick in the last couple of years; this may be partly due to cyclical factors.

The relative constancy of the depreciation to GDP ratio can be explained by making use of the following identity:

$$Depreciation/GDP = (Depreciation/Capital\ Stock) \times (Capital\ Stock/GDP)$$

In the United States the capital stock to GDP ratio (in current prices) has moved in the opposite direction to the depreciation rate so the depreciation to GDP ratio is relatively flat. In the United Kingdom, for variants BEA and ONS1, the depreciation rate has fallen slightly (Chart 25), while the capital stock to GDP ratio has remained fairly stable. Hence the depreciation to GDP ratios for these variants has fallen slightly. For variant ICT3, in the latter part of the period, the depreciation rate has risen as has the capital stock to GDP ratio. These effects reinforce each other so that the depreciation to GDP ratio rises.

(47) GDP in current prices (ONS code: ABML) includes an estimate of capital consumption (ONS code: NQAE). To get the ratio of nominal depreciation to GDP consistent, we first subtract the ONS estimate of capital consumption from the denominator of the ratio and then add back our measure of depreciation. This way the depreciation figures in the numerator and denominator are consistent. Note that NQAE is inclusive of dwellings, whereas our measures are for whole economy minus dwellings. So when calculating the ONS ratio we make a further adjustment for capital consumption in dwellings (ONS code: EXCT). For example, the depreciation GDP ratio for Variant ICT3 is DEPICT3/(ABML - (NQAE - EXCT) + DEPICT3) and for the ONS ratio it is (NQAE-EXCT)/ABML.

Chart 25

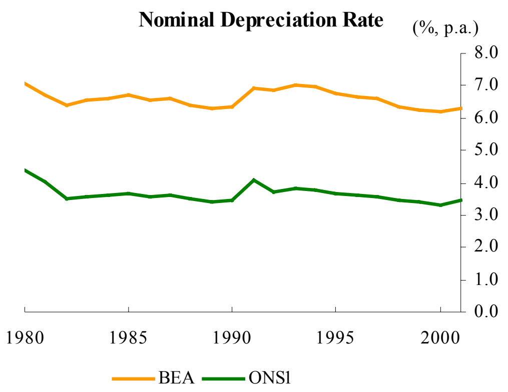


Chart 26

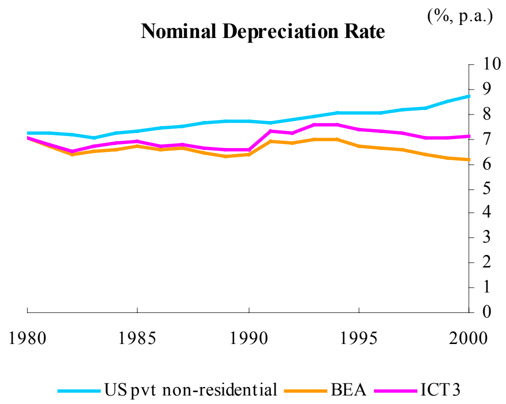


Chart 27

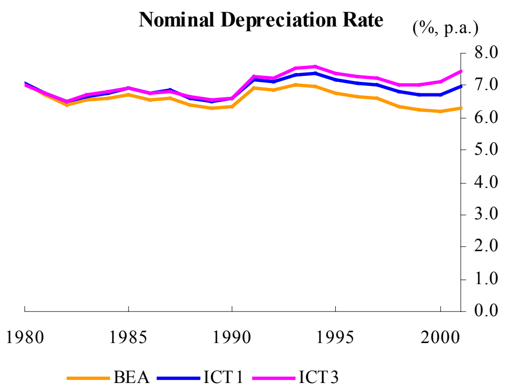


Chart 28

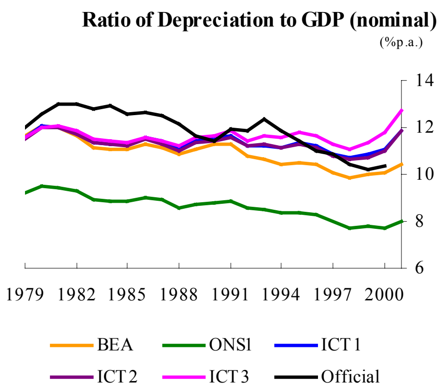
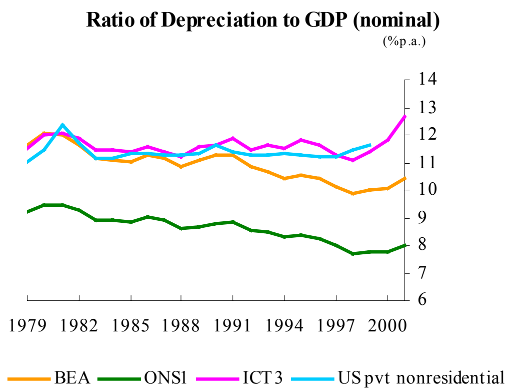


Chart 29



Does cyclical scrapping affect the results qualitatively? The answer is no. As is evident from Charts 30-32 which show the aggregate depreciation rate for variant ICT3, the temporary nature of cyclical scrapping affects the aggregate rate (nominal or real) for only a very short time. The same holds true for the other variants.

Chart 30

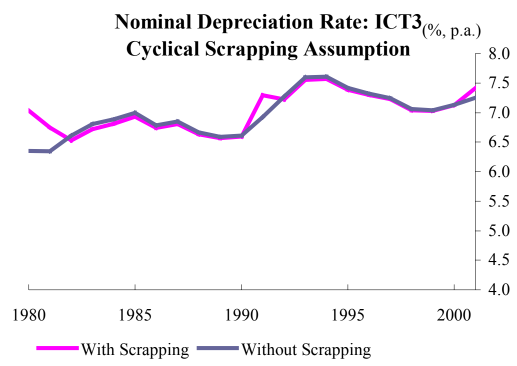


Chart 31

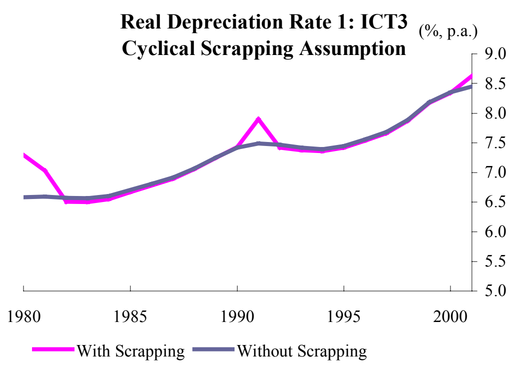
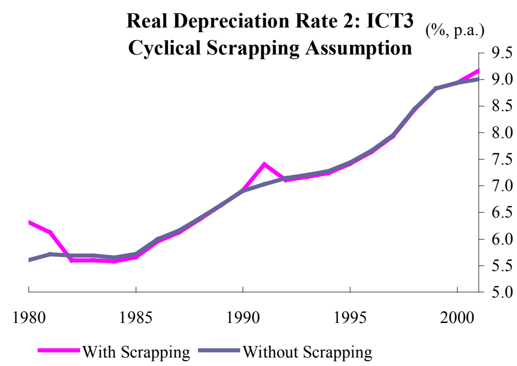


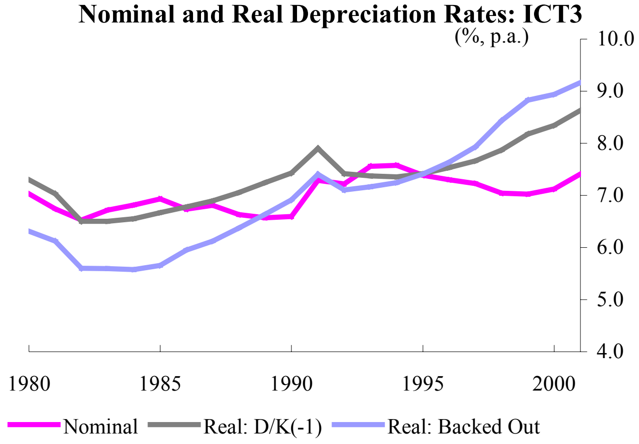
Chart 32



Note that the overall pattern of the real depreciation rate is different in Charts 31 and 32 depending on the method used to calculate it. Section 3 discussed two methods of calculating the real aggregate depreciation rate. Real measure one calculated it as a ratio of real depreciation to real capital stock and real measure two backed it out from the capital accumulation identity equation. Because we use chain-linked data, there is no reason why real measure one should equal real measure two. Chart 33 presents nominal and both real measures of aggregate depreciation rate for variant ICT3. The caveat stated in the conclusion to the subsection on aggregate depreciation in Section 3 should be kept in mind: we should use the real definitions of the aggregate depreciation rate with caution.

Chart 33

Nominal and Real Depreciation Rates: ICT3



Profitability

In the process of estimating the VICS, we have had to calculate the nominal, post corporation tax, rate of return on capital for a range of assumptions about depreciation rates and asset composition. This rate of return has some independent interest as a measure of profitability. So we would like to know how sensitive it is to our assumptions.

The nominal rate of return is defined as aggregate nominal profit, net of depreciation and net of corporation tax, as a percentage of the nominal value of the aggregate capital stock at the beginning of the year. To recall: all fixed assets (excluding dwellings) plus inventories are included in the aggregate capital stock when the rate of return is estimated and the stock itself is net of depreciation. We can convert this rate of return into a real rate by subtracting from it a measure of inflation. For the latter we use the growth rate of the GDP deflator.[48]

Charts 34 and 35 show the nominal and real rates for three sets of assumptions: BEA, ONS1 and ICT3. The rates of return turn out to be remarkably insensitive to the depreciation rate assumptions. The explanation is that, while a higher depreciation rate raises the amount of depreciation on a given stock of an asset, thus lowering net profit, it simultaneously reduces the estimated stock. It turns out that these two effects roughly cancel out. In fact, the rate of return is lowest with the low depreciation rates (and long asset lives) of the ONS1 assumption. Over the most recent period 1995-2000, the real rate averaged 6.67% per annum under ICT3, 7.04% under BEA and 5.90% under ONS1.

The real rate of return is highly cyclical, falling sharply in the two major recessions of 1980-82 and 1990-92 and even turning negative in the latter. Over 1979-2000, it averaged 4.0% to 4.7% per annum (depending on the assumptions). If this appears low, recall that our measure is for the whole economy and so includes health, education and government, sectors where profit is not the main concern.

[48] The GDP deflator is gross value added at current basic prices divided by gross value added at 1995 basic prices (ONS codes ABML ÷ ABMM).

Chart 34

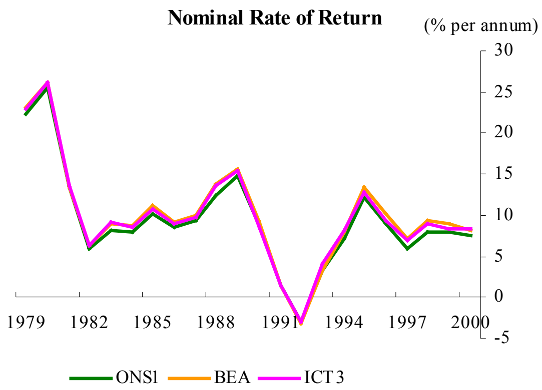
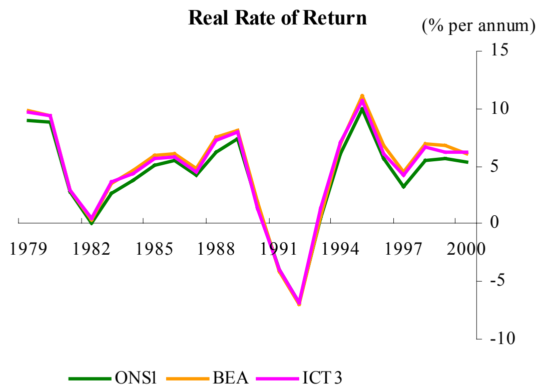


Chart 35



5 Conclusions

We have set out an integrated framework for estimating the wealth stock, the VICS, and depreciation (capital consumption). The resulting estimates are consistent both theoretically and empirically. In this framework the distinction between decay, which describes how the services of a capital asset change as the asset ages, and depreciation, which describes how the prices of assets of different ages vary, is crucial. We have seen that the estimation process is greatly simplified if we adopt the assumption that depreciation is geometric, since then the rates of decay and depreciation are equal. We have reviewed the evidence for geometric depreciation. Unfortunately, there is little direct evidence for the United Kingdom. Most studies relate to the United States and even here the evidence is far from complete. But it is fair to say that the geometric assumption is found to fit the facts quite well. Hence, it has been officially adopted as the 'default' assumption in the US NIPA.

The paper has also considered whether the geometric assumption is appropriate for assets like computers. Computers do not suffer much from physical wear and tear, but nevertheless have very short lives due to what is usually called 'obsolescence'. We found that, in principle, our framework encompasses obsolescence. A properly specified hedonic regression, applied to panel data on new and second-hand asset prices, can estimate the true rate of depreciation, even in the presence of obsolescence. But if an empirically important quality variable is omitted from the regression, the estimate of depreciation will be biased. If quality is improving the bias will be positive, ie the estimated rate of depreciation will be too high. The depreciation rates used by the BEA in the US NIPA are based on studies of new and second-hand asset prices. Since these studies were not always able to control fully for (generally rising) quality, it may be that the BEA rates are overstated. In the specific case of computers, we have argued that the US evidence supports a geometric rate of about 30% rather than the 40% used by the BEA.

We have accordingly adopted the geometric assumption in our empirical work for the United Kingdom. Because of the uncertainty about asset lives and the pattern of depreciation in the United Kingdom, we have calculated wealth and VICS measures under a range of assumptions. We have tested the sensitivity of our results in three main ways. First, we compare results using both US and UK assumptions about asset lives. Second, we compare results based on a comparatively coarse breakdown of assets into four types only, with results derived from a more detailed breakdown in which computers and software are distinguished separately. Third, we compare the effect of US versus UK price indices for computers and software. Our results are for the whole economy and all fixed assets excluding dwellings, for the period 1979 Q1-2002 Q2. Our main findings for wealth and VICS are as follows:

1. Using the conventional, four fold breakdown of assets into buildings (excluding dwellings), plant and machinery, vehicles, and intangibles, we find that the growth rates of wealth and the VICS are insensitive to variations in depreciation rates.
2. By the nature of the measure, the level of the VICS will be insensitive to depreciation rates if the growth rate is. This is because the real level of the VICS equals the nominal level in the base year, whatever the assumption about depreciation. This nominal level is just aggregate profit in current prices. So before and after the base year the estimated level will be affected by different assumptions about depreciation only to the extent that the growth rates are affected. However, no such restriction applies to wealth. In fact, the *level* of wealth is found to be quite sensitive to variations in depreciation rates.
3. Still sticking with the conventional asset breakdown, wealth and VICS grew at similar rates over the period as a whole. In the 1990s, the gap between the two measures widens a bit, with the growth rate of the VICS higher by about 0.1 percentage points per quarter.
4. The effect on the estimates of separating out computers and software is quite complex. First, with these assets separated out, much larger differences appear between the growth rates of VICS and wealth, of the order of 0.2-0.4 percentage points per quarter. Second, comparing results with and without computers and software being separated, we find that separating them out tends to reduce the growth rate of wealth, while not necessarily increasing that of the VICS. But when we use the set of assumptions which are closest to US methods (eg US price indices for computers and software plus the ‘times 3’ adjustment to the level of software investment), the growth rate of the VICS is raised by 0.2 percentage points per quarter, relative to the VICS with computers and software included in with other assets.
5. The VICS tends to be more volatile than wealth when volatility is measured by the standard deviation of the growth rate. Using US methods for computers and software tends to raise volatility.

For some purposes, growth rates are more important than levels. If so, these results suggest that the empirically important issue is the measurement of investment in computers and software. It is common ground that the relative price of these assets has been falling, so in principle it is correct to separate them out explicitly. The conclusions about the growth rates of both VICS and wealth turn out to be very sensitive to the price index used for computers and to the correction made to the level of software investment.

For other purposes, eg measuring Tobin’s Q, levels matter. In these cases, there is no substitute for further research into asset lives. But it turns out that profitability, the real rate of return on capital, is not sensitive to the asset life assumptions.

We have also estimated aggregate depreciation (capital consumption) for the same range of assumptions. We have studied the sensitivity of the aggregate depreciation rate and of the ratio of depreciation to GDP to the assumptions, and compared our estimates with ones derived from official data. On theoretical grounds we prefer to measure both these ratios in current prices. Our findings here are as follows:

1. Using the conventional asset breakdown and our assumptions about depreciation rates at the asset level, there is no tendency for the aggregate depreciation rate to rise over the last two decades. In other words, the asset mix has not been shifting towards more rapidly depreciating assets like plant and machinery, vehicles or intangibles.
2. Separating out computers and software has less effect than one might have expected. Even using US methodology raises the aggregate rate by only about 1 percentage point to 7% in 2000 and again there is no sign of an upward trend. The reason is that even by 2000 the share of computers and software in wealth was only about 4%. By contrast and on a comparable basis, the aggregate depreciation rate in the United States has trended smoothly upwards since 1980, to reach nearly 9% in 2000. This illustrates the much greater scale of ICT investment in the United States.
3. The assumptions about asset lives have a large impact on the estimated ratio of depreciation to GDP. The official measure taken from the national accounts has been drifting down fairly steadily since 1979. In 2001 it stood at 8%. Using US asset lives and the conventional asset breakdown, the ratio was over 10% in the same year. Separating out ICT assets and using US methods, the ratio rises to nearly 13%, similar to the ratio in the United States. Interestingly, in neither country is there any upward trend in the ratio, except perhaps in the past couple of years. The reason is that though the quantity of high-depreciation assets has been growing faster than GDP, this has been offset by their falling price.

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Appendix A: Proofs of propositions in the text

A.1. Proof that geometric depreciation implies geometric decay and of the converse

The price of a new asset at the end of period t-1 is:

$$p^A_{t-1,0} = \sum_{z=0}^{\infty}\left[p^K_{t+z,z} \Big/ \prod_{\tau=0}^{z} (1+r_{t+\tau}) \right] \qquad \textbf{(49)}$$

This is the same as equation **(2)** of the main text except that we have made the possibility of an infinite life explicit. Analogously, the price of an asset of age s at the same time is:

$$p^A_{t-1,s} = \sum_{z=0}^{\infty}\left[p^K_{t+z,s+z} \Big/ \prod_{\tau=0}^{z} (1+r_{t+\tau}) \right] \qquad \textbf{(50)}$$

The relationship between the rental prices is given by **(14)**, repeated here for convenience:

$$p^K_{t,s} / p^K_{t,0} = f_s / f_0$$

Similarly,

$$p^K_{t+z,s+z} / p^K_{t+z,z} = f_{s+z} / f_z, \qquad z = 0,1,2,...$$

Dividing **(50)** by **(49)**, the ratio of the two asset prices is:

$$p^A_{t-1,s} / p^A_{t-1,0} = \sum_{z=0}^{\infty}\left[(f_{s+z}/f_z)\cdot p^K_{t+z,z} \cdot h_z \right] / \sum_{z=0}^{\infty}\left[p^K_{t+z,z} \cdot h_z \right] \qquad \textbf{(51)}$$

where we have put:

$$h_z = 1 / \prod_{\tau=0}^{z} (1+r_{t+\tau})$$

for the discount factor.

Geometric decay implies geometric depreciation
If decay is geometric, then $f_{s+z} / f_z = (1-d)^s$. That is, the decay factor depends only on the difference in the ages, not the absolute ages. The ratio of the asset prices is then:

$$p^A_{t-1,s} / p^A_{t-1,0} = (1-d)^s$$

since we can factor the decay factor out from the summation in **(51)**. Hence depreciation is at the geometric rate d, ie $\delta = d$.

Geometric depreciation implies geometric decay
If depreciation is geometric at rate δ, then from **(51)**

$$(1-\delta)^s = \sum_{z=0}^{\infty}\left[(f_{s+z}/f_z)\cdot p^K_{t+z,z} \cdot h_z \right] / \sum_{z=0}^{\infty}\left[p^K_{t+z,z} \cdot h_z \right], \qquad s = 0,1,2,...$$

The only term on the right-hand side of this equation which depends on *s* is the factor f_{s+z} / f_z. Hence the only way that this equation can be satisfied for all values of *s* is if f_{s+z} / f_z is independent of the absolute ages and depends only on the difference in ages, so that we can write $f_{s+z} / f_z = \phi_s$ say. Then we can factor it out of the summation to obtain:

$$(1-\delta)^s = \phi_s = (1-d)^s$$

Hence decay is at the geometric rate δ, ie $d = \delta$.

A.2. Proof that assets with proportionally high rental prices receive more weight in a VICS than in a wealth measure

According to equation **(28)**, the relation between rental prices and asset prices is:

$$p_{it}^K = T_{it}\left[r_t \cdot p_{i,t-1}^A + \delta_i \cdot p_{it}^A - (p_{it}^A - p_{i,t-1}^A)\right]$$

$$= T_{it}\left[r_t + \delta_i(1+\pi_{it}) - \pi_{it}\right] p_{i,t-1}^A$$

where $\pi_{it} = (p_{it}^A - p_{i,t-1}^A) / p_{i,t-1}^A$, the rate of growth of the asset price. Now define the ratio of the rental price to the asset price by:

$$\rho_{it} = p_{it}^K / p_{i,t-1}^A$$

To simplify notation, normalise all asset prices to unity: $p_{i,t-1}^A = 1, \forall i$. The VICS weights are now:

$$w_{it} = \rho_{it} K_{it} / \sum_{i=1}^m \rho_{it} K_{it}$$

and the weights in the wealth measure are:

$$v_{i,t-1} = K_{it} / \sum_{i=1}^m K_{it}$$

Then we have the following:

Proposition If asset *i* has a rental asset price ratio which is higher than the asset-value-weighted average for all assets, then its weight in the VICS is higher than its weight in the wealth measure. In symbols,

$$\text{If } \rho_{it} > \sum_{j=1}^m v_{jt}\rho_{jt}, \text{ then } w_{it} > v_{i,t-1}$$

Proof Assume the contrary: $w_{it} \le v_{i,t-1}$. Then from the definition of the weights:

$$\frac{\rho_{it} K_{it}}{\sum_{j=1}^{m} \rho_{jt} K_{jt}} \leq \frac{K_{it}}{\sum_{j=1}^{m} K_{jt}}$$

which implies that:

$$\rho_{it} \leq \sum_{j=1}^{m} \left(\frac{K_{jt}}{\sum_{j=1}^{m} K_{jt}} \right) \cdot \rho_{jt} = \sum_{j=1}^{m} v_{jt} \rho_{jt}$$

This is a contradiction, so the proposition is proved.

A.3 Proof of proposition about real depreciation rate, δ_t^R

Proposition
Suppose as in the text that there are two assets. Both assets are growing at constant rates but the first asset is growing more rapidly and also has a higher depreciation rate. Technology is Cobb-Douglas, so that the current price share of each asset in the aggregate capital stock is constant. Consider the ratio of real depreciation to the real capital stock. Then (a) under chain-linking, this ratio will rise without limit so that eventually it exceeds the rate on the higher of the two individual depreciation rates; (b) with a fixed base index, the ratio approaches the higher of the two rates asymptotically.

Proof
(a) Chain-linking
This result will be proved using a Törnqvist chain index, which is generally a good approximation to a Fisher chain index. Let D_{it} be real depreciation on asset type *i* in period *t*:

$$D_{it} = \delta_i A_{i,t-1}$$

The Törnqvist chain index of aggregate real depreciation is:

$$\Delta \ln D_t = \sum_{i=1}^{m} w_i \Delta \ln D_{it} = \sum_{i=1}^{m} w_i \Delta \ln A_{i,t-1}$$

where:

$$w_i = \delta_i p_{it} A_{i,t-1} / \sum_{i=1}^{m} \delta_i p_{it} A_{i,t-1}, \text{ all } t\,.$$

are the constant-over-time wealth shares. The Törnqvist chain index of wealth is:

$$\Delta \ln A_{t-1} = \sum_{i=1}^{m} v_i \Delta \ln A_{i,t-1}$$

where:

$$v_i = p_{it} A_{i,t-1} / \sum_{i=1}^{m} p_{it} A_{i,t-1}, \text{ all } t$$

Now specialise these definitions to the case of two assets. Asset 1 has a high depreciation rate, asset 2 a low one ($\delta_1 > \delta_2$). Suppose that the growth rates of the assets are constant but that asset 1 grows more rapidly: $g_1 > g_2$ where the g_i are the growth rates (defined as log differences). So in this case:

$$\Delta \ln D_t = w_1 g_1 + w_2 g_2 \text{ and } \Delta \ln A_{t-1} = v_1 g_1 + v_2 g_2$$

where $w_1 + w_2 = v_1 + v_2 = 1$. The wealth shares (v_1, v_2) and the shares in aggregate nominal depreciation (w_1, w_2) are related by

$$\frac{w_1}{w_2} = \frac{\delta_1 p_{1t} A_{1,t-1}}{\delta_2 p_{2t} A_{2,t-1}} = \frac{\delta_1}{\delta_2} \frac{v_1}{v_2}$$

Now since by assumption $\delta_1 > \delta_2$, it follows that $w_1 /(1 - w_1) > v_1 /(1 - v_1)$ and so that $w_1 > v_1$. Consequently, $\Delta \ln D_t > \Delta \ln A_{t-1}$. The difference between these two growth rates is constant, so the ratio of real depreciation to real wealth rises without limit. Eventually, the aggregate depreciation rate must exceed the individual rates.

(b) Fixed-base indices

With fixed-base indices, we can set prices in the base year equal to 1, so that aggregate depreciation is:

$$D_t = \delta_1 A_{1,t-1} + \delta_2 A_{2,t-1}$$

and the capital stock is:

$$A_t = A_{1t} + A_{2t}$$

The ratio of depreciation to the capital stock is:

$$\frac{\delta_1 A_{1,t-1} + \delta_2 A_{2,t-1}}{A_{1,t-1} + A_{2,t-1}}$$

If asset 1 is growing faster, then this ratio approaches δ_1 as *t* goes to infinity.

Appendix B: Data appendix

Investment

The following table shows the annual and quarterly, seasonally adjusted investment series we have used, together with the ONS codes for the current and constant price series.

Table B.1 ONS codes for gross investment

	Quarterly series, sa		*Annual series*	
Asset type	*Current prices*	*1995 prices*	*Current prices*	*1995 prices*
1. Other buildings and structures	EQED	DLWT	DLWS(49)	DLWQ
2. Transport equipment	TLPX	DLWL	DLWZ	DLWJ
3. Other machinery and equipment and cultivated assets	TLPW	DLWO	DLXI	DLWM
4. Intangible fixed assets	TLPK	EQDO	DLXP	EQDT
5. Changes in inventories	Not used	CAFU	not used	ABMQ

Real asset stocks

(a) Annual We calculated annual asset stocks from 1963 onwards, using starting stocks for end-1962 generated as in earlier work (Oulton (2001a)) and employing equations **(31)** and **(32)**. The stock of inventories used the value at the end of 2000 in 1995 prices (from the Quarterly National Accounts, 2nd quarter 2001) as the basis. The stock in other years was then calculated from the changes in inventories series.

(b) Quarterly The quarterly investment series start in 1965 Q1. We used the annual capital stock model to generate a starting stock for each asset at the end of 1964 Q4. Then for the four fixed assets, the stock of each asset was accumulated from 1965 Q1 onwards using the quarterly investment series (see above), employing equation **(26)**. The quarterly stock of inventories was calculated in the same way as the annual stock.

The depreciation rates are based on those used by the BEA in the US NIPA, described in Fraumeni (1997), and are shown in Table B.2. The BEA rates themselves are at a more disaggregated level; the rates in the table are averages of these more detailed rates. The average rate for plant and machinery in the United States is now considerably higher than 13%, due to the rise in importance of computers and software; the 13% figure was appropriate for the 1970s. But since later we make special provision for computers and software, the 13% figure has been retained.

(49) This current price annual investment series for buildings does not include transfer costs. In our calculations we have added transfer costs [DFBH] to this series for 1965-98. From 1999 onwards, the annual values are calculated as the sum of the quarterly values for that year. The quarterly series in current and constant prices and the annual series in constant prices for buildings investment include transfer costs already.

Table B.2 Depreciation rates (per cent per annum)

Asset	BEA lives (Variant BEA)	ONS lives (Variant ONS1)
1. Other buildings and structures	2.5	100*0.90/79 = 1.14
2. Other machinery and equipment and cultivated assets	13.0	100*1.65/29 = 5.69
3. Transport equipment	25.0	100*1.853/9= 20.59
4. Intangible fixed assets	31.5	31.5
5. Inventories	0.0	0.0

Asset prices

The asset price of each asset type except inventories is derived as an implicit deflator: the current price investment series divided by the constant price investment series. For inventories, we used the price index for all manufacturing, excluding duties [PNVQ], from 1974 onwards and, prior to then, the price index including duties [PLLU]. The annual asset prices formed part of the estimation of the rental price weights. The annual or quarterly asset prices can also be used to convert asset stocks to nominal terms.

Tax/subsidy factor

The tax/subsidy factors (T_{it}) were kindly supplied by Rod Whittaker (HMT). They are annual. There are separate factors for plant and machinery, industrial buildings, and vehicles. We used the tax factor for plant and machinery for intangibles, computers and software as well. The tax factor for inventories was set equal to 1.

Rental prices

To calculate the rental prices and hence the weights for each asset type in the VICS, we include inventories and the fixed assets and use these to solve for first, the nominal rate of return, and next, for the rental prices. Because dwellings are excluded, the appropriate profit total is the aggregate gross operating surplus minus what should be attributed to ownership of dwellings. Total profit is therefore measured as gross operating surplus [ABNF] less actual and imputed rentals on housing [ADFT+ADFU].

The estimated rental price weights were unsatisfactory in a number of ways. First, the rental weight for buildings plunged in 1974 and 1980 in an implausible manner, while that for plant and machinery rose sharply. These spikes were removed by making 1974 the average of 1973 and 1975, and 1980 the average of 1979 and 1981, for these assets. Second, the rental weight for inventories was extremely volatile. This was dealt with by fitting a time trend to the weight and substituting the predicted for the actual values. The weights were then adjusted so that they continued to sum to 1. Finally, we took a two-year moving average of the weights.

Appendix C: A software investment series for the United Kingdom

This appendix updates the series for software investment presented in Oulton (2001a).[50] The nominal series which we use for variant ICT3 is constructed from official data but is then multiplied by three for reasons discussed in Oulton (2001a): this is referred to as the 'times 3' adjustment. The nominal data are deflated by the US price index for pre-packaged software as published in Parker and Grimm (2000), adjusted for changes in the sterling-dollar exchange rate. Pre-packaged software is about a third of the total in the United States, the other two components being custom and own account software. The pre-packaged component is the only one for which a true price index exists. Hence we use this to deflate all software.

C.1 Revising the existing current-price series for software investment

Total software investment in current prices is available from the Supply and Use Tables published by the ONS for 1989-2000. Oulton (2001a) extrapolated this data series backwards to 1964 using information from various input-output tables.

The ONS publish investment series for five categories of assets: dwellings, other buildings and structures, other machinery and equipment (OME), vehicles and intangibles. If we want to treat software as a separate category then we need to know in which of the above five categories it is nested so that we can adjust the figures accordingly to avoid double counting. Despite what a casual reading of Tables 6.2 and 6.3 and paragraph 6.15 of *Concepts, Sources and Methods* might suggest, only part of software investment is included in intangible investment; the rest is in the 'other machinery and equipment' category. So aggregate data for both other machinery and equipment and intangible assets have to be adjusted to avoid double counting.

Using data kindly supplied by the ONS (for 1970 onwards), we can extract a series for that part of software investment that is in the intangible asset category. The part of software investment in other machinery and equipment can then be calculated as a residual. In the calculations, we first calculate software (OME) for 1989-2000 by subtracting software (intangibles; NPJG) from the total. For 1970-88, software (OME) is calculated as a proportion of software (intangibles) where the proportion is that of software (OME) divided by software (intangibles) in 1989. The total for 1970-88 is then calculated as the sum of software (OME) and software (intangibles).[51]

C.2 Updating the current-price series for software investment to 2001

The raw data only go so far as 2000 but we require an extended series to 2001. The total and software (intangibles) are extended by assuming that the growth rates in 2000 are the same as that of plant and machinery. Software (OME) is then calculated as the residual.
The annual nominal data for 1970-2001 are converted to quarterly nominal data using the procedure described below. For 2002 Q1-2002 Q2, the quarterly nominal series are extrapolated using the quarterly growth rate of plant and machinery.

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(50) Table B.2, page 59, without the 'times 3' adjustment.

(51) In current prices, *software (OME) + software (intangibles) = software (total).* We use the proportion of *software (OME)/software (intangibles)* in 1989 to extrapolate the *software (OME)* series backwards first; the total is then calculated as the sum of the components. This ensures non-negativity of the sub-categories of total software investment.

C.3 Constant-price series for software investment and the associated investment price deflator

The basic procedure is as follows. For each series (computers and software) we start out with annual nominal data on investment, its associated quarterly investment price deflator[52] and an 'indicator' quarterly series that behaves like the economic variable in question.

1. The annual nominal data are converted into quarterly nominal data by using the Chow-Lin interpolation procedure.[53] This procedure requires an input indicator series. It uses the movement of the indicator series to interpolate the quarterly series from the annual investment series.
2. The quarterly nominal series is then deflated by the quarterly investment price deflator to get the quarterly real investment series. Both the ONS and the BEA take the annual price in the base year to be equal to 1, but this annual price is an arithmetic mean of the quarterly prices. This means that the sum of the quarterly real values will not, in general, equal the annual real value (calculated by dividing the annual nominal value by the annual price deflator =1) in the base year. To be consistent with the ONS, we have to ensure base year consistency in the annual values (ie, nominal = real). Hence, the quarterly series are scaled in the ratio of the base year annual nominal value to the base year 'sum of quarterly reals' so that after the scaling the sum of the quarterly real series is equal to the annual nominal value in the base year.
3. The annual real series for years other than the base year are calculated similarly by summing the scaled quarterly real values.

The implication for the implicit price deflators is that they no longer equal the 'published series'; in fact, they are the published series times the scaling factor. This obviously matters for the levels but (i) has no effect on the growth rates and (ii) the scaling factor is very small for the series that we have (for example, using UK computer prices, the scaling factor is 0.003%). The scaling also means that the annual price deflator in the base year (calculated as the arithmetic mean of the quarterly price deflators) is no longer exactly equal to one but the difference is negligibly small. Table C.1 summarises the procedure.

The indicator series used to convert the annual computer and software investment series into their respective quarterly counterparts is the (quarterly) nominal investment in (total) plant and machinery.

(52) If quarterly price deflator series are not available, we have used the annual deflator series and kept the value in each quarter of the year equal to the annual value (eg, done for US computer and software prices).
(53) See Chow and Lin (1971). A RATS subroutine developed by John Frain (Central Bank of Ireland) is used for the interpolation.

Table C.1

	Frequency of Data	
Units of Measurement	Annual Nominal Chow-Lin Procedure using ‘Indicator’ series	Annual Real
	Quarterly Nominal Quarterly price deflator	Scale and Sum Quarterly Real

Assuming that the deflators are the same across software components (in OME, in intangibles and total), we calculate the constant price series by dividing the current price series by the software investment deflator.

Table C.2

Software Investment in the United Kingdom

	In Intangible Asset Category	*In Other Machinery and Equipment Category*	*Total*	*Implied Deflator*[54]	*In Intangible Asset Category*	*In Other Machinery and Equipment Category*	*Total*[55]
	(£ million, current prices)					(constant prices)	
1970	17	12	29	9.77	2	1	3
1971	21	15	36	8.17	3	2	4
1972	24	17	41	7.05	3	2	6
1973	31	22	53	6.97	4	3	8
1974	38	27	65	6.51	6	4	10
1975	49	35	84	6.57	7	5	13
1976	66	48	114	7.29	9	7	16
1977	79	57	136	6.90	11	8	20
1978	99	72	171	5.05	20	14	34
1979	128	93	221	4.09	31	23	54
1980	160	116	276	3.24	49	36	85
1981	182	132	314	3.47	52	38	90
1982	233	169	402	3.75	62	45	107
1983	283	205	488	3.89	73	53	125
1984	345	250	595	3.91	88	64	152
1985	455	329	784	3.64	125	90	215
1986	533	386	919	2.82	189	137	326
1987	591	428	1019	2.28	259	187	446
1988	709	513	1222	1.94	365	264	629
1989	864	625	1489	1.76	491	355	846
1990	1030	781	1811	1.40	734	556	1290
1991	1101	700	1801	1.35	816	520	1336
1992	1180	647	1827	1.08	1091	597	1688
1993	1235	827	2062	1.21	1018	682	1700
1994	1286	1146	2432	1.09	1182	1056	2238
1995	1320	1535	2855	1.00	1320	1535	2855
1996	1548	1483	3031	0.95	1624	1553	3177
1997	1703	1317	3020	0.83	2051	1586	3637
1998	2206	2071	4277	0.76	2910	2733	5643
1999	2417	2016	4433	0.76	3196	2665	5861
2000	2801	1922	4723	0.81	3456	2374	5830
2001	2790	1914	4704	0.83	3351	2300	5651

Note Components in the above tables may not add up to the totals exactly due to rounding.

(54) Because of the scaling required to convert quarterly real data to annual real data, the implied deflator is not exactly equal to sterling equivalent of the Parker and Grimm (2000) annual deflator (conversion from dollars done using the sterling exchange rate (ONS code: AJFA)).

(55) The components add to the total because we are using the same investment price deflator.

Appendix D: Backing out non-computer investment from total investment

D.1 Introduction

The series for investment in plant and machinery published by the ONS includes computers. This appendix considers how to reconstruct the series that ONS would have arrived at, had they decided to exclude computers. For this purpose we use ONS methods and rely entirely on ONS data.

The ONS publishes data on total investment in 'Other machinery and equipment' (*OME*), which includes computers, in both constant and current prices. We also have ONS data on a component of *OME*, computer investment, for the period 1976-2000. The nominal computer series derives from the Input-Output Supply and Use Tables for 1989 onwards; prior to 1989, our series is constructed from the various input-output tables, with missing years interpolated. The real computer series is the nominal series deflated by the official PPI for computers (ONS code PQEK). We want to derive investment in *OME* excluding computers (*OMEXC*). Obviously, there is no problem in doing this in current prices by simple subtraction, but how to do it in constant prices is not so straightforward.

D.2 The chain-linked solution

For the period 1994 to the present, the ONS uses 1995 prices. So for this period we can indeed calculate *OMEXC* by subtracting computer investment in 1995 prices from total *OME* in 1995 prices. But prior to 1994 the ONS used different weights: successively 1990, 1985, 1980 and 1975 prices as we go back in time. In other words the ONS does not use a fixed base index but instead a type of chain index in which the weights are periodically updated (about every five years in practice).[(56)]

For each of the periods over which the weights are constant, the index of *OME* investment is in effect constructed by the ONS as follows:

$$QOME = w \cdot QCOMP + (1-w) \cdot QOMEXC \qquad \textbf{(52)}$$

where *QOME* is the index of total investment, set equal to 1 in the base year, *QCOMP* is a similar index for computer investment, *QOMEXC* is the index for other plant and machinery, and w is the weight for computers. This weight is the nominal share of computer investment in the total in the base year (successively 1975, 1980, 1985, 1990 and 1995). We can find the *QOME* index for (say) 1984 relative to 1985 by dividing *OME* investment in 1995 prices for 1984 by *OME* investment in 1995 prices for 1985. This works because rebasing to 1995 prices does not change growth rates for earlier periods. We can calculate the *QCOMP* index similarly. Therefore, for each period covered by a singe base, we can solve this equation for *QOMEXC*:

$$QOMEXC = [QOME - w \cdot QCOMP]/(1-w) \qquad \textbf{(53)}$$

(56) For this reason saying that such indices are 'in 1995 prices' or 'in constant prices' is potentially misleading. It might be better to say that these series are in 'chained 1995 pounds' (copying the BEA usage of 'chained 1996 dollars').

We can then link all these fixed-base index numbers together, so that we have a type of chain index which covers the whole period. This chain index can be referenced to any year we choose, without changing its growth rate. Suppose we choose 1995 as the reference year when the index takes the value 1. Then we can multiply the chain index in each year by the nominal value of *OMEXC* in 1995, thus obtaining *OMEXC* in constant 1995 prices.

To illustrate the process, consider the following imaginary data for an *OMEXC* index calculated using equation **(53)**. Here the base periods are assumed to be periods 1 and 4 and the link period is 3.

Table D.1
Illustrative calculation of chain index from sequence of fixed-base indices

	Fixed base index		*Chain index*	
Period	*Base: Period 1*	*Base: Period 4*	*Reference: period 1*	*Reference: period 4*
1	1.000	—	1.000	0.888
2	1.050	—	1.050	0.932
3	1.070	0.950	1.070	0.950
4	—	1.00	1.126	1.000
5	—	1.10	1.239	1.100

When the reference period for the chain index is period 4, the value of the index in eg period 2 is calculated as (1.05 ÷ 1.07) x 0.95 = 0.932.

D.3 Non-additivity

In general, chain indices are non-additive: the components do not necessarily sum to the total. In other words, if we add OMEXC in 1995 prices to COMP in 1995 prices, the result will not be equal to OME in 1995 prices, except for the period 1994 to the present when the ONS has used 1995 as the base. If the component (computers) which is growing more rapidly has a falling relative price, as is the case here, then the ONS's chain index of OME grows more rapidly than the sum of the components before the base year, here 1995. This implies that the level of the ONS's chain index for OME is less than the sum of the components in constant prices in all years prior to 1994:

$$OMEXC + COMP > OME$$

or

$$OMEXC > OME - COMP$$

In other words, our (and implicitly the ONS's) chain-based estimate of the non-computer component will be greater than the estimate one would obtain by naively subtracting computer investment from total investment, in all years prior to 1994. Consequently, the growth rate of OMEXC will be *less* than the growth rate of the naïve (fixed-base) estimate prior to 1995 (since the levels are the same from 1994 onwards).

This is illustrated in Charts D.1 and D.2. The level of the naïve, fixed-base index is 25% below that of the chain index of OMEXC in 1976. Between 1976 and 1994 the fixed-base index grew at 2.34% per annum, while the chain index grew at only 0.76% per annum. Putting it another way, the sum of computer investment (COMP) and the new chain series of the total excluding computers (OMEXC) exceeds the actual total of OME investment by a growing amount as we go back further in time. By 1976 the sum of the two components exceeds the total by 33%. But to reiterate, this is just a consequence of chain-linking in the form used up to now by the ONS. That the difference between the two types of estimate is so large reflects the substantial fall in the relative price of computers which occurred over this period. If we had used the more rapidly falling US price index for computers, instead of the UK one, the difference would have been even more striking. But our aim here is to construct the series for non-computer investment which the ONS would have arrived at themselves had they chosen to do so, so we employ their methods and data.

Chart D.1

Comparison of chain and fixed base indices of *OMEXC:* levels

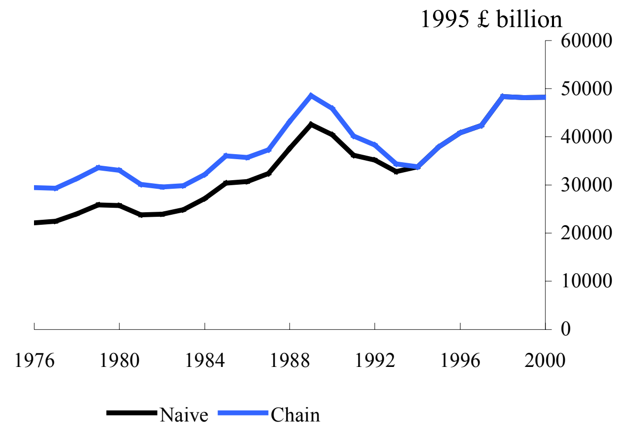
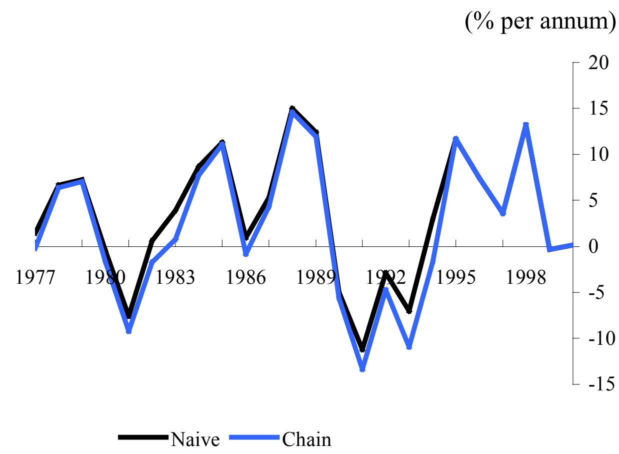


Chart D.2

Comparison of chain and fixed base indices of *OMEXC:* growth rates



Appendix E: Shares in wealth and profits and average growth rates of stocks, 1995 Q1-1999 Q4

Table E.1: Shares in nominal wealth (W) and profits (P) by asset: 1995 Q1-1999 Q4

(per cent)

Variant	Buildings		Plant		Vehicles		Intangibles		Computers		Software	
	W	P	W	P	W	P	W	P	W	P	W	P
BEA	70	42	24	45	4	10	2	3	--		--	
ONS1	65	35	31	51	3	10	1	3	--		--	
ICT1	69	43	24	38	4	10	1	3	1	5	--	
ICT2	69	43	24	38	4	10	1	3	1	6	--	
ICT3	68	40	24	36	4	10	2	3	1	6	2	6

Table E.2: Average growth rates of asset stocks: 1995 Q1-1999 Q4 (per cent per quarter)

Variant	Buildings	Plant	Vehicles	Intangibles	Computers	Software
BEA	0.68	1.52	0.81	0.36	--	--
ONS1	0.72	1.29	0.70	0.36	--	--
ICT1	0.68	0.52	0.81	0.36	6.66	--
ICT2	0.68	0.52	0.81	0.36	8.94	--
ICT3	0.68	0.47	0.81	-0.04	8.94	4.94

Table E.3: Variance of Shares in nominal wealth (W) and profits (P) by asset: 1979 Q1-2002 Q2

(per cent squared)

Variant	Buildings		Plant		Vehicles		Intangibles		Computers		Software	
	W	P	W	P	W	P	W	P	W	P	W	P
BEA	4.2	106.2	3.2	76.8	0.1	5.2	0.08	0.5	--		--	
ONS1	6.2	258.5	5.4	212.8	0.1	5.7	0.04	0.4	--		--	
ICT1	4.9	120.7	3.9	86.4	0.1	5.2	0.08	0.5	0.1	2.8	--	
ICT2	5.0	123.8	3.9	85.0	0.1	5.1	0.08	0.5	0.1	2.8	--	
ICT3	5.6	129.4	3.6	81.6	0.1	5.2	0.09	0.3	0.1	2.6	0.4	3.5

Table E.4 Variance of growth rates of asset stocks: 1979 Q1-2000 Q2 (per cent squared)

Variant	Buildings	Plant	Vehicles	Intangibles	Computers	Software
BEA	0.02	0.36	1.14	0.98	--	--
ONS1	0.01	0.17	0.77	0.98	--	--
ICT1	0.02	0.27	1.14	0.98	2.40	--
ICT2	0.02	0.27	1.14	0.98	6.98	--
ICT3	0.02	0.27	1.14	1.06	6.98	7.14

Chart E.1

Asset Stock Growth Rates: Buildings

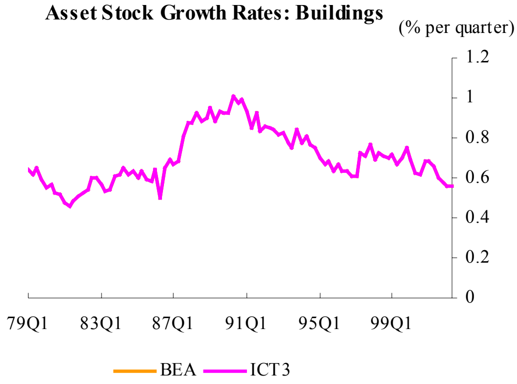


Chart E.2

Asset Stock Growth Rates: Plant

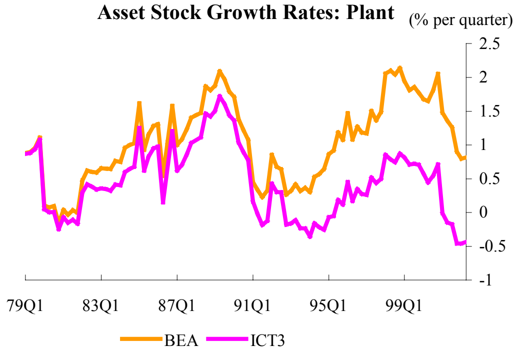


Chart E.3

Asset Stock Growth Rates: Computers

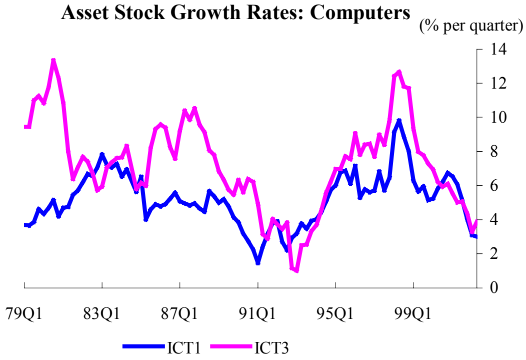


Chart E.4

Asset Stock Growth Rates: Software

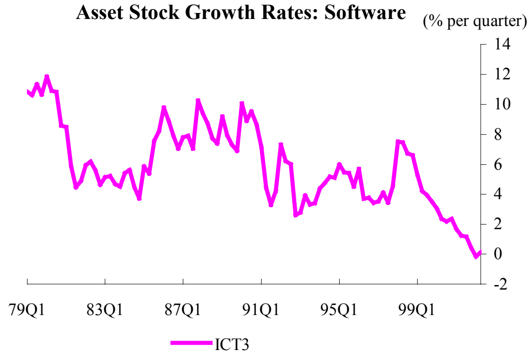


Chart E.5

Share of Buildings in Wealth

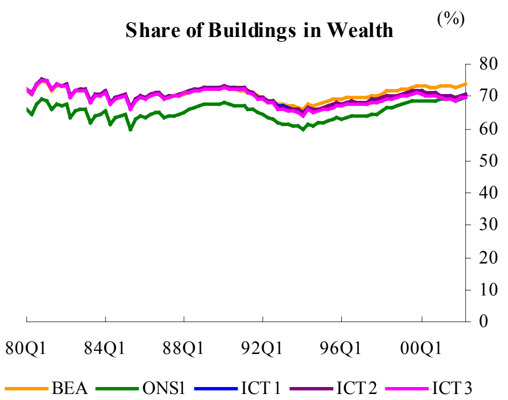


Chart E.6

Share of Buildings in Profits

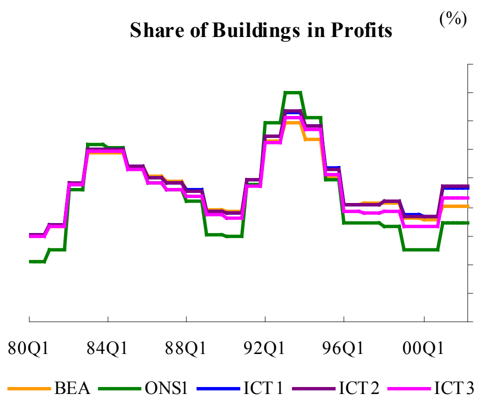


Chart E.7

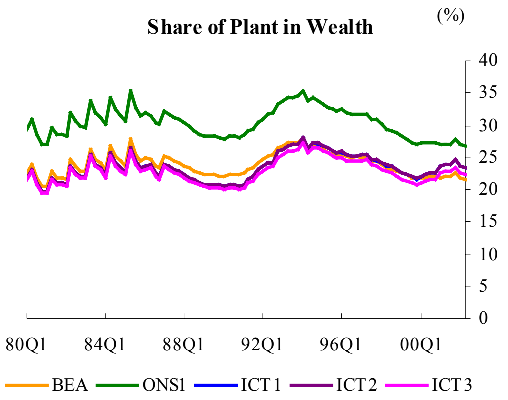


Chart E.8

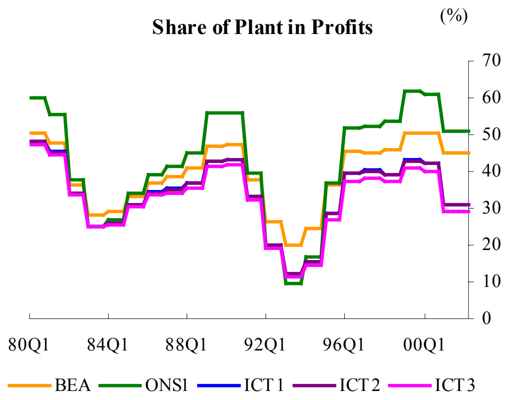


Chart E.9

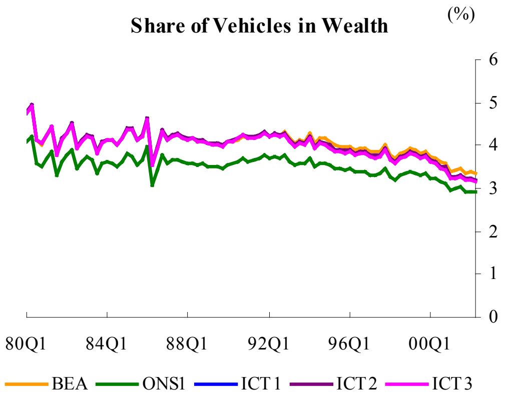


Chart E.10

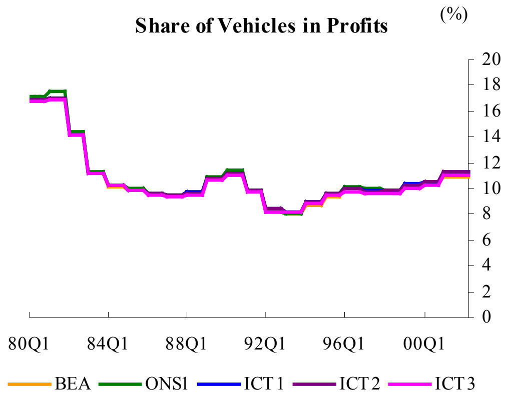


Chart E.11

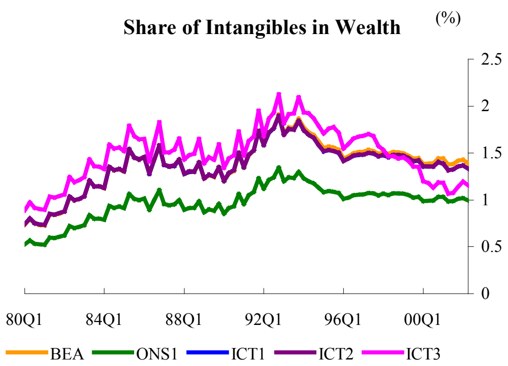


Chart E.12

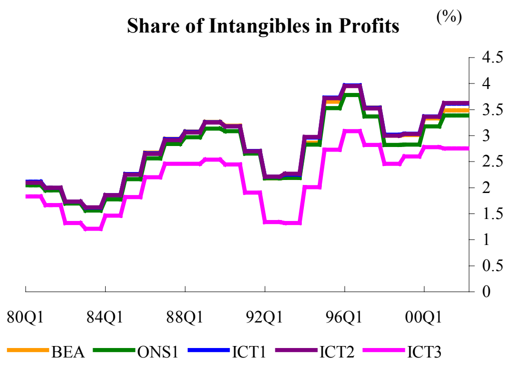


Chart E.13

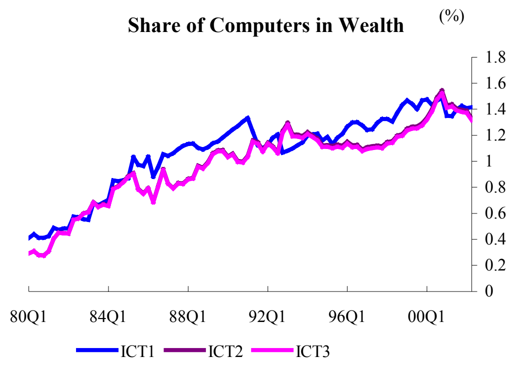


Chart E.14

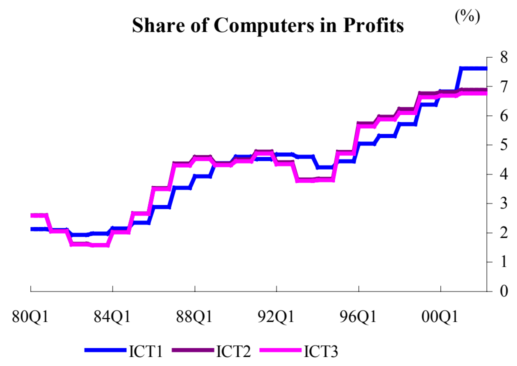


Chart E.15

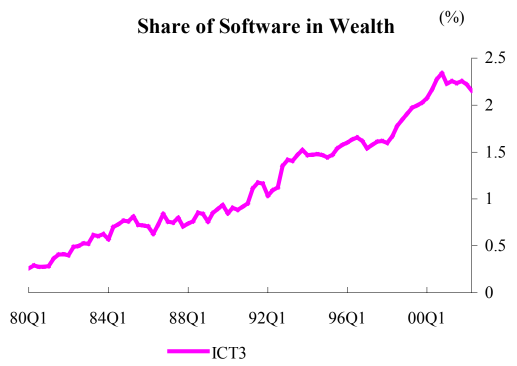
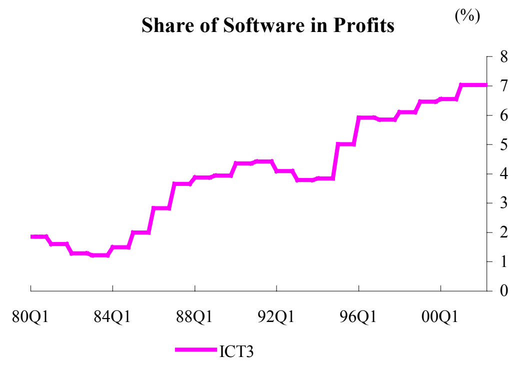


Chart E.16



Explaining the Investment Boom of the 1990s

Stacey Tevlin
Karl Whelan*

March 7, 2000

Abstract

Real equipment investment in the United States has boomed in recent years, led by soaring investment in computers. We find that traditional aggregate econometric models completely fail to capture the magnitude of this recent growth—mainly because these models neglect to address two features that are crucial (and unique) to the current investment boom. First, the pace at which firms replace depreciated capital has increased. Second, investment has been more sensitive to the cost of capital. We document that these two features stem from the special behavior of investment in computers and therefore propose a disaggregated approach. This produces an econometric model that successfully explains the 1990s equipment investment boom.

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1 Introduction

The behavior of equipment investment in the current U.S. expansion has been remarkable. Growth in real equipment investment over the period 1992-98 averaged 11.2 percent per year, exceeding all other seven-year intervals in the post-War era.[1] This development has been of great macroeconomic importance: The investment boom has underpinned the continuing strength of U.S. aggregate demand and has probably also had important supply-side effects, perhaps playing a role in the unusual late-cycle acceleration in labor productivity.

In this paper, we examine whether existing time series models can explain the astounding behavior of equipment investment in the 1990s. We demonstrate that they cannot. Although we examine the traditional, accelerator-style models that previous investment "horserace" studies have found best fit the data, we find that they completely fail to capture the magnitude of the 1990s investment boom.[2] We show that the models' breakdown stems from an important element of investment growth in the 1990s—the surge in real investment in *computing equipment*. Our analysis of the behavior of computer investment reveals two features that, though crucial to the investment boom of the 1990s, are ignored by standard aggregate models. We demonstrate that a *dis*aggregated approach, which models investment in computing and non-computing equipment separately, successfully explains the behavior of investment in the 1990s.

The first feature that we document is the sharp increase in the average rate of depreciation in the 1990s. Most econometric models assume a constant depreciation rate and thus a stable relationship between the change in the capital stock and the level of investment. Since the optimal capital stock is a function of the level of output and the cost of capital, this also implies a stable link between investment and changes in output and the cost of capital. However, the increasing rate of depreciation in the 1990s broke this link: Firms needed to invest more to sustain a given level of the capital stock. We show that the increase in the depreciation rate was due to a shift in the composition of capital towards computers,

[1] All figures in this paper refer to 1992-based National Income statistics rather than the 1996-based figures published in October 1999. The econometric models in our paper use capital stock data, and revised capital stocks will not be published until Spring 2000.

[2] For earlier evaluations of competing investment models, see Clark (1979) and Bernanke, Bohn, and Reiss (1988). Oliner, Sichel, and Rudebusch (1995) compare accelerator-style models with models based on Euler equations or Tobin's Q.

which depreciate more rapidly than other types of equipment. Aggregate models do not capture this phenomenon, because, by definition, they ignore compositional mix-shifts.

The second feature that we examine is the role of the cost of capital. The rising average depreciation rate suggests the need to separately model net investment in computing and non-computing equipment. Doing so reveals an important pattern. Computer investment is very sensitive to the cost of capital, far more so than investment in non-computing equipment. As a result, rapid declines in computer prices played a crucial role in generating the investment boom of the 1990s. This result contrasts sharply with most of the empirical literature on aggregate investment, which typically finds very little response to cost variables. We provide a plausible explanation for the different estimates of cost-of-capital elasticities that we observe: Firms respond more to shocks perceived as permanent than to those perceived as transitory, and shocks to computer prices usually result from technological innovations that are unlikely to be reversed.

We conclude that the special behavior of equipment investment in the 1990s resulted from the substantial impact of rapid computer price declines on capital accumulation, and the consequent need for higher rates of replacement investment. A simple disaggregated approach, which separately models net and gross investment for computing and non-computing equipment is capable of explaining the recent behavior of investment.

The paper is organized as follows. Section 2 reviews the traditional econometric models and documents their poor empirical performance in the 1990s. Section 3 examines the increase in the average rate of depreciation and its role in the breakdown of the conventional models. Section 4 discusses why capital accumulation may respond more to the persistent component of the cost of capital than to the less persistent component. Section 5 presents our econometric analysis and documents the performance of our approach in tracking the behavior of aggregate gross investment in the 1990s. Section 6 concludes.

2 Traditional Investment Models and Their Recent Failure

Traditional models of investment start with a theory relating the optimal frictionless capital stock, K_t^*, to the production technology and factor prices. If firms could costlessly adjust the capital stock, they would always set $K_t = K_t^*$. However, the sluggish behavior of the capital stock suggests that there are costs associated with adjustment. The traditional neo-Keynesian investment models used simple *ad hoc* specifications of the effects of adjustment costs, the most common being the partial adjustment approach, which assumed that firms move part of the way towards their optimal frictionless stock each period. Formulating this relationship in terms of the logarithm of the capital stock and using lower case letters to denote the log of variables, the partial adjustment equation is

$$\Delta k = (1-\lambda)\left(k_t^* - k_{t-1}\right) \tag{1}$$

which can be re-written as:

$$k_t = \lambda k_{t-1} + (1-\lambda) k_t^* \tag{2}$$

Applying repeated substitution to equation (2) gives an equivalent representation for the capital stock, this time as an infinite distributed lag function of past k_t^*'s:

$$k_t = \sum_{r=0}^{\infty} (1-\lambda)\lambda^r k_{t-r}^* = \sum_{r=0}^{\infty} \gamma_r k_{t-r}^* \tag{3}$$

This has been turned into an empirical investment equation by taking the following steps. First, the infinite distributed lag suggested by the partial adjustment theory is replaced with a finite approximation, usually about 8 to 12 quarters. Second, the equation is differenced to turn it into a net investment equation:

$$\Delta k_t = \sum_{r=0}^{N} \gamma_r \Delta k_{t-r}^* + \epsilon_t \tag{4}$$

Of course, if the capital stock adjustment equation is correctly specified, then this differencing step is not necessary. However, the traditional literature largely pre-dated cointegration methods and used stationarity-inducing transformations as a precaution against spurious regressions. In our empirical work, we will examine this issue formally.

This equation is operationalized by assuming a form for k_t^*. Specifying a CES production function, K_t^* is proportional to $\frac{Y_t}{{C_t}^{\sigma}}$, where Y_t is output, C_t is the cost of capital, and σ is the elasticity of substitution between capital and labor. Taking logs of K_t^* we get

$$\Delta k_t = \sum_{r=0}^{N} \gamma_r \Delta y_{t-r} - \sigma \sum_{r=0}^{N} \gamma_r \Delta c_{t-r} + \epsilon_t \tag{5}$$

Since $\sum_{r=0}^{\infty} \gamma_r = 1$, the sum of the coefficients on output should approximately equal one while the coefficients on the cost of capital should sum to the elasticity of substitution, σ. These sums have an intuitive interpretation since they describe the predicted long-run response of the capital stock to permanent unit shocks to output and the cost of capital.

Models of the form of equation (5) have been estimated by Bernanke, Bohn, and Reiss (1988). However, this approach, which describes the determination of the capital stock, only gives us a model of *net* investment. For macroeconomists interested in business cycle modelling and forecasting, the variable of interest is *gross* investment, which includes both the change in capital stock and the replacement of depreciated capital. Most empirical models assume a constant average rate of depreciation and estimate an equation for gross investment. In this case, approximating the log-difference of the capital stock with the growth rate, we get

$$\Delta k_t \approx \frac{\Delta K_t}{K_{t-1}} = \frac{I_t}{K_{t-1}} - \delta \tag{6}$$

where δ is the depreciation rate. This gives an equation for gross investment

$$\frac{I_t}{K_{t-1}} = \delta + \sum_{r=0}^{N} \gamma_r \Delta y_{t-r} - \sigma \sum_{r=0}^{N} \gamma_r \Delta c_{t-r} + \epsilon_t \tag{7}$$

We will label this regression the "traditional model". Note that this approach estimates the depreciation rate as the intercept in the $\frac{I_t}{K_{t-1}}$ regression.

Previous empirical implementations of this model, estimated on data prior to the 1990s, have found that it provides a fairly good description of the cyclical behavior of investment. Indeed, Oliner, Rudebusch, and Sichel (1995) have shown that models of this form provide superior forecasting performance to popular alternative specifications based on Euler equations or Tobin's Q. This is not to say that these models are without problems. For instance, despite microeconomic evidence that the elasticity of substitution is close to one, regressions usually reveal a small and often insignificant role for the cost of capital. Indeed, comparisons of forecasting power have often favored the pure accelerator formulation ($\sigma = 0$) over models including the cost of capital. Summarizing these results, Chirinko's comprehensive 1993 survey concluded that "on balance, the response of investment to prices tends to be quite small and unimportant relative to quantity variables."

Our estimation of equation (7) confirms these previous results, revealing a small long-run cost-of-capital elasticity of -0.34. (Our data are described in Appendix A.) However, Figure 1, which shows the in-sample fit, reveals a far more serious problem. The model fails completely to capture the 1990s' increase in investment relative to the capital stock. After 1991, the model underpredicts by larger and larger amounts, with these residuals principally offset by large negative residuals over the early part of the sample.[3] By 1997:4, the actual level of investment relative to the capital stock is 7 percentage points higher than can be explained by the model; this translates into a 31 percent error on the level of investment. A Chow test for parameter stability confirms that the model has gone off track in the 1990s, with the null hypothesis of stable coefficients resoundingly rejected.[4] In the rest of the paper, we explore the reasons for the traditional model's complete breakdown in the 1990s.

3 The Unstable Aggregate Depreciation Rate

The most obvious simplifying assumption made in the derivation of the traditional model is that the average rate of depreciation is constant. We can easily check the validity of this assumption by solving for the aggregate depreciation rate obtained from re-arranging the perpetual inventory equation ($K_t = (1 - \delta_t) K_{t-1} + I_t$) to get:

$$\delta_t = \frac{I_t - \Delta K_t}{K_{t-1}} \tag{8}$$

Figure 2 shows this series as the solid line (rather obscurely labeled "Using Chain-Weight Investment and Capital", for reasons that will become apparent in a moment). It shows that the aggregate depreciation rate has not been constant, but has increased substantially in recent years, rising from 0.13 in 1989 to 0.16 in 1997.

[3]This is also true for popular alternative versions of this equation such as the Jorgenson "neoclassical" model and an augmented version that includes cash flow. Models based on Tobin's Q, although predicting strong investment over the last few years of our sample, also do not track the behavior of $\frac{I_t}{K_{t-1}}$ in the 1990s particularly well.

[4]The figure also reveals a problem reported in previous horserace papers. Even when residuals are small during the middle part of our sample, they tend to be positively autocorrelated. We believe this is due to the finite-lag approximation to the true infinite-lag capital stock adjustment formula, equation (3). When λ is large (and empirical estimates suggest it is), this approximation will omit autocorrelated terms. Our regressions in Section 5 are not based on the finite-lag approximation.

The main cause of this uptrend is straightforward. Different types of equipment depreciate at different rates and Oliner (1989, 1994) has shown that computers depreciate significantly faster than other types of equipment. The National Income and Product Accounts (NIPA) capital stocks used in our analysis are constructed under the assumption of separate, constant, depreciation rates for each of 27 underlying equipment categories, with the depreciation rate for computers taken directly from Oliner's research.[5] Thus, it should come as no surprise that the recent explosion in computer investment has led to an increase in the average rate of depreciation for total equipment. Before we move on to discuss the implications of a varying pace of depreciation for econometric modelling, we need to note a surprising pattern in our calculated series for the aggregate depreciation rate.

3.1 A Depreciation Puzzle

While we had expected that the high rates of computer investment would have raised the aggregate depreciation rate in the 1990s, we were surprised to find that this was not just a recent phenomenon but rather an acceleration of a long-running trend. In fact, our calculated series for the aggregate depreciation rate *doubles* over 1965-1997. The magnitude of this apparent mix-shift seems very large, particularly as it suggests that variations in the average rate of depreciation have had an important effect on aggregate gross investment throughout the past 30 years, something not found by previous researchers. The solution to this puzzle turns out to be a change in the NIPA methodology for constructing real aggregates.

Since 1996, all NIPA real expenditure aggregates, including real GDP, have been derived using a Fisher chain-aggregation methodology.[6] Since 1997, real capital stock aggregates have been constructed using the same methodology. Rather than aggregating all quantities according to their base-year prices, as in the traditional Laspeyres index, the growth rate of a chained aggregate reflects a mix of old and new prices. Given a series of quantities and prices for n goods, $q_i(t)$ and $p_i(t)$, the gross growth rate for the Fisher chain-aggregate quantity is defined as:

$$G(t) = \sqrt{\frac{\sum_{i=1}^{n} p_i(t)\, q_i(t)}{\sum_{i=1}^{n} p_i(t)\, q_i(t-1)} \frac{\sum_{i=1}^{n} p_i(t-1)\, q_i(t)}{\sum_{i=1}^{n} p_i(t-1)\, q_i(t-1)}} \tag{9}$$

[5] See Katz and Herman (1997) for a description of the NIPA stocks.

[6] See Landefeld and Parker (1997) for a discussion of this methodology.

In the base-year (1992 in our data), all price indexes are set equal to one and the level of each aggregate is set equal to its nominal value. For all subsequent and previous years, the real level series are simply "chained" forward and backwards using the Fisher-aggregation growth rates. For NIPA aggregate real equipment investment and stocks, the Fisher chain procedure aggregates 27 component series.

This chain aggregation procedure helps to reduce biases due to valuing goods at prices that become irrelevant once we move away from the base year. However, a complexity it introduces is that *the level of the constructed real aggregate is no longer the additive sum of its real components*, with this lack of additivity being most evident when there are large relative price shifts within a bundle of goods (as is the case with the equipment bundle because of the substantial declines in the price of computing equipment). This non-additivity invalidates the calculation of the aggregate depreciation rate. To illustrate, consider the following simple example.

There are two types of capital, A and B. Suppose now that the aggregate capital stock is constructed according to the traditional Laspeyres fixed-weight formula. In this case, the real aggregates for investment and the capital stock are the simple sum of their real components: $I^{FW} = I^A + I^B$ and $K^{FW} = K^A + K^B$. It is easy to use this fact to show that the aggregate depreciation rate is a weighted average of the two underlying depreciation rates, with the weights given by the real quantities for the two stocks:

$$\delta_t^{FW} = \frac{I_t^{FW} - \Delta K_t^{FW}}{K_{t-1}^{FW}} = \delta^A \left(\frac{K_{t-1}^A}{K_{t-1}^A + K_{t-1}^B} \right) + \delta^B \left(\frac{K_{t-1}^B}{K_{t-1}^B + K_{t-1}^A} \right) \tag{10}$$

Note, however, that the strict additivity of the fixed-weight formula was necessary to obtain this weighted-average expression for the aggregate depreciation rate. Once this additivity breaks down, only in the base year can we interpret the depreciation rate calculated from equation (8) as a weighted average; this is because in the base year all real series are equal to their nominal counterparts and so for this year additivity does hold. In fact, as we show in Appendix B, moving away from the base year, the aggregate depreciation rate calculated from equation (8) with chain-weighted data differs systematically from a weighted-average depreciation rate, displaying a long-run upward trend even in the absence of mix shifts towards faster depreciating equipment.

The explanation for this result is fairly subtle; a full derivation is available in Appendix B. However, the intuition is as follows. The growth rate of a chain-weighted aggregate effectively equals a weighted-average of the growth rates of its components, where the weights are given by the components' nominal shares. It turns out that when real investment in one type of capital grows faster than others because its relative price is declining, then the nominal share of this type of capital in investment will be higher than its nominal share in the capital stock, implying that the aggregate for real investment grows faster than the aggregate for the real capital stock. As a result, the ratio of the level of real aggregate investment to the level of the real aggregate capital stock, will trend upwards. Hence, the series calculated from equation (8) will also trend upwards.

To demonstrate the effect on the aggregate depreciation rate of the change in aggregation methodology, we constructed fixed-weight aggregates for equipment investment and the equipment capital stock by adding up the underlying real series for the 27 equipment investment categories. We then calculated a depreciation rate for these fixed weight series, exactly as in equation (10). This fixed-weight depreciation rate series is shown as the thick dashed line on Figure 2 (labeled "Using Fixed-Weight Investment and Capital"). It rises steeply over the past few years but increases only very slightly prior to the 1990s, remaining for most of the sample in the range of 0.13, the value most commonly used in studies that construct equipment stocks from a constant aggregate depreciation rate. In contrast, the corresponding series for chain-weighted data climbs steadily from the mid-1960s on. Thus, although the increasing aggregate depreciation rate in the 1990s mainly reflects a composition shift, the uptrend evident prior to the 1990s is mainly an artifact of chain aggregation.[7]

Since mix-shifts towards equipment-types that are faster depreciating and declining in relative price can explain the substantial rise over time in our perpetual-inventory estimate of the aggregate depreciation rate, an obvious question is whether removing computing equipment (which depreciates rapidly and has the largest price declines) will result in a stable depreciation rate. The final series on Figure 2, the thin dashed line (labeled "Non-Computing Equipment, Chain-Weight Investment and Capital"), tells us that the answer is: Almost. This series was calculated by applying equation (8) to newly-calculated chain-aggregates for investment and capital stock for all equipment except computers, and it shows a very slow and modest upcreep over time.[8]

[7] Because previous research in this field used the old fixed-weight data, this explains why other researchers did not note this curious pattern.

[8] Appendix A describes how we calculated these aggregates for non-computing equipment.

3.2 Implications for Aggregate Investment Modelling

Returning to the recent failure of the traditional model, we have seen that the actual depreciation rate required to convert aggregate net investment ($\Delta k_t = \frac{I_t}{K_{t-1}} - \delta_t$) into $\frac{I_t}{K_{t-1}}$ is not a constant that can be proxied by the intercept, as is assumed when we directly estimate equation (7), but in fact has been rising rapidly in recent years. If our aim is a stable econometric model, then a solution to this problem is to instead directly model the behavior of net investment by estimating equation (5). Figure 3 illustrates how this step radically improves in-sample fit. It compares the residuals from our estimation of the gross investment ($\frac{I_t}{K_{t-1}}$) model, equation (7), with the residuals from estimation of the net investment ($\frac{I_t}{K_{t-1}} - \delta_t$) model, equation (5). Once we do not have to account for the variations in the aggregate depreciation rate, we no longer have residuals that trend up over time and the recent net investment residuals, though still positive and relatively large, are not historically unprecedented.

This is something of a hollow victory, however, if our ultimate goal is a model of gross investment expenditures. Worse still, these aggregate models cannot explain the source of the increasing aggregate depreciation rate–the explosion in net investment in computing equipment. A complete model of gross investment expenditures in the 1990s must account for the different behavior of investment in computing and non-computing equipment. In the next section, we present an alternative to the partial adjustment model that provides intuition for such an approach by illustrating why computer prices may have a different effect on investment than other elements of the cost of capital.

4 Cost of Capital Shocks and Capital Accumulation

Empirical tests of the traditional models that we have focused on thus far find only a small role for price variables. Therefore, they imply that rapidly declining computer prices have had little impact on investment in computers. The sheer magnitude of the increase in computer investment in recent years suggests that this may be incorrect. Consider now an alternative theoretical approach, previously presented by Nickell (1979) and Kiyotaki and West (1996), that explains why computer price declines may affect capital accumulation more than other shocks.

The models we have looked at thus far rely on very simple modelling of the effects of adjustment costs. An alternative is to explicitly model the implications of adjustment costs for an optimizing firm with rational expectations. To capture only the essential features of the investment problem, we use a quadratic approximation to the underlying profit function: Changes in the capital stock and deviations from the frictionless optimal stock both lead to costs which increase according to a simple quadratic function. For a given expected path of k^*, firms choose the current capital stock to solve

$$Min \;\; E_t \left[\sum_{m=0}^{\infty} \theta^m \left\{ \left(k_{t+m} - k^*_{t+m} \right)^2 + \alpha \left(k_{t+m} - k_{t+m-1} \right)^2 \right\} \right] \tag{11}$$

where θ is the firm's discount rate.

The model's first-order conditions are:

$$E_t \left[-k_{t+1} + \left(1 + \frac{1}{\theta} + \frac{1}{\alpha\theta} \right) k_t - \frac{1}{\theta} k_{t-1} - \frac{1}{\alpha\theta} k^*_t \right] = 0 \tag{12}$$

Letting L be the lag operator, F be the lead operator, and using the fact that the characteristic equation $x^2 - \left(1 + \frac{1}{\theta} + \frac{1}{\alpha\theta}\right) x + \frac{1}{\theta} = 0$ has two roots such that one root (λ) is between zero and one while the other equals $\frac{1}{\theta\lambda}$, this can be re-expressed as

$$E_t \left[-\left(F - \lambda \right) \left(F - \frac{1}{\theta\lambda} \right) L k_t - \frac{1}{\alpha\theta} k^*_t \right] = 0$$

Implying a solution

$$k_t = \lambda k_{t-1} + \frac{\lambda}{\alpha} E_t \left[\sum_{n=0}^{\infty} \left(\theta\lambda \right)^n k^*_{t+n} \right] \tag{13}$$

An intuitive re-formulation of this equation that illustrates the model's fundamental property, comes from using the fact that $\frac{\lambda}{\alpha} = (1 - \lambda)(1 - \theta\lambda)$ (which comes from re-arranging the characteristic equation). Making this substitution we get

$$\Delta k_t = (1-\lambda)\left(k_t^{**} - k_{t-1}\right) \tag{14}$$

where

$$k_t^{**} = (1-\theta\lambda)\, E_t \left[\sum_{n=0}^{\infty} (\theta\lambda)^n \, k_{t+n}^{*}\right] \tag{15}$$

Thus, each period, the log of the capital stock adjusts towards the moving target, k_t^{**}, which is a weighted average of expected future k_t^*'s. It can be shown that λ depends positively on α, implying that higher adjustment costs lead to a slower speed of adjustment towards k_t^{**}.[9]

The model is completed by a specification of the process for k_t^*. Profit maximization (using a generalized CES production function) will give us a first order condition: $k_t^* = \eta_t + y_t - \sigma c_t$, where y and c are as before and η_t summarizes the effects of capital-biased technological change. To give a concrete example of what η_t means, the stock of computing capital may tend to rise independently of output and the cost of capital if the structure of production changes in ways that facilitate increased usage of computers.

To implement this model empirically, we need to specify time series processes for output and the cost of capital. Let $y_t = \phi(L)\, y_{t-1} + \epsilon_t$ and $c_t = \pi(L)\, c_{t-1} + \nu_t$ where ϕ and π are m-th order distributed lag polynomials and ϵ_t and ν_t are white noise. Given these processes, we can solve for the effects of output and the cost of capital on k_t^{**} in terms of empirically observable variables by using the following formula of Hansen and Sargent (1980):

$$E_t \left[\sum_{n=0}^{\infty} (\theta\lambda)^n \, y_{t+n}\right] = \kappa(L)\, y_t \tag{16}$$

where

$$\kappa(L) = \frac{1}{1-\phi(\theta\lambda)} \left[1 + \sum_{k=1}^{m-1} \left(\sum_{r=k+1}^{m} (\theta\lambda)^{r-k} \phi_r\right) L^k\right] \tag{17}$$

Similarly, letting

$$\mu(L) = \frac{1}{1-\pi(\theta\lambda)} \left[1 + \sum_{k=1}^{m-1} \left(\sum_{r=k+1}^{m} (\theta\lambda)^{r-k} \pi_r\right) L^k\right] \tag{18}$$

[9] This type of capital stock process can also be derived from more general assumptions about technology and adjustment costs: See Auerbach (1989).

the process for the capital stock is now

$$k_t = \lambda k_{t-1} + (1-\lambda)(1-\theta\lambda)(\kappa(L)\, y_t - \sigma\mu(L)\, c_t) + \hat{\eta}_t \tag{19}$$

where $\hat{\eta}_t$ depends on a weighted average of current and future values of the capital-biased technological change term.

Suppose now we estimate equation (19). The technology-bias variable, $\hat{\eta}$, cannot be observed, so this ends up in the error term. Thus, our estimating equation is

$$k_t = \alpha + \lambda k_{t-1} + \sum_{i=0}^{N} \beta_i y_{t-i} + \sum_{i=0}^{N} \gamma_i c_{t-i} + u_t \tag{20}$$

where the model predicts that

$$\begin{aligned} \beta(L) &= (1-\lambda)(1-\theta\lambda)\,\kappa(L) \\ \gamma(L) &= \sigma(1-\lambda)(1-\theta\lambda)\,\mu(L) \\ \alpha + u_t &= \hat{\eta}_t \end{aligned}$$

Equation (20) bears a close resemblance to the capital stock equation under partial adjustment. However, the coefficients on y and c now depend on the variables' own time-series processes and the discount rate, θ, as well as on the underlying production technology and the adjustment speed, λ. Specifically, consider the long-run elasticities with respect to y and c, defined as the sum of coefficients on these variables divided by $(1-\lambda)$. These values depend positively on the *persistence* of the explanatory variables. In Appendix C, we show that if c is an $I(1)$ series, then $(1-\theta\lambda)\,\mu(1) = 1$. But, if c is an $I(0)$ series, then this term is less than 1, and will be approximately zero if c is white noise. The reason for this result is intuitive: Firms are less likely to react to shocks to the "frictionless optimal" stock that they perceive as being temporary than to shocks perceived to be permanent.[10]

[10] Note that these are *conditional* elasticities, not long-run impulse responses of a multiple equation system: They describe the behavior of the capital stock conditional on the paths of output and the cost of capital. This contrasts with the work of Kiyotaki and West (1996). They have also noted that this model can allow the capital stock to have different elasticities with respect to output and the cost of capital. However, their empirical implementation imposed the assumption that the cost of capital was an $I(1)$ series, thus ruling out this possibility. Their implementation of this model instead focused on long-run impulse responses of the (k, y, c) system. Their finding of smaller long-run impulse responses to shocks to c comes from their estimated process for c being a less persistent $I(1)$ process than the $I(1)$ process for output (for instance although both are $I(1)$ processes, $y_t = 1.5y_{t-1} - 0.5y_{t-2} + \epsilon_t$ implies larger impulse responses than $y_t = 0.5y_{t-1} + 0.5y_{t-2} + \epsilon_t$). It does not come from smaller *conditional* elasticities for k with respect to c than with respect to y.

In light of these results, it is informative to examine the persistence properties of the cost of capital for computing and non-computing equipment. We define the cost of capital according to the standard Hall-Jorgenson rental rate formula:

$$C_t = P_t \left(R_t + \delta - \frac{\dot{P}_t}{P_t} \right) \left(\frac{1 - ITC - \tau * DEP}{1 - \tau} \right) \tag{21}$$

where P_t is the price of capital relative to the price of output, R_t is the real interest rate, ITC is the investment tax credit, DEP is the present value of depreciation allowances per dollar invested, and τ is the marginal corporate income tax rate.

Expressed in logs, the cost of capital is the sum of two series–the relative price of capital, and the non-relative-price component, which measures the tax-adjusted gross required rate of return on investment. As Figure 4 shows, these two components affect the computer and non-computer cost of capital series in very different ways. The upper panels show that the computer cost of capital is highly non-stationary, exhibiting continuous rapid declines as a result of the remarkable pattern of falling purchase prices. The lower panels show that the relative stability of the non-computer cost of capital comes from a combination of an uneven decline in the relative price of this equipment and a choppy pattern for the non-price component.

Even looking within specific categories, the cost of capital combines components that appear to have very different persistence properties. For instance, the relative price of computers appears to be a very persistent series; the relative price of non-computing equipment seems to have a downward trend, although one that is less dominant than for computers; the non-price components for both variables seem to be relatively stable, mean-reverting series. More formal econometric characterizations of the persistence of these series, using simple autoregressions and unit root tests, confirm the intuition implied by these graphs. These tests suggest that the relative price series for both computing and non-computing equipment almost certainly have unit roots, while the non-relative price components appear more likely to be stationary series.

There are also good *economic* reasons to believe that the price and non-price components of the cost of capital have different persistence properties. The pattern of declining relative prices for equipment comes from technological innovations in the equipment-producing industries, and it seems likely that once prices have fallen as a result of innovations, these price reductions will be permanent. In contrast, real interest rates will, in the long-run, be related to the marginal productivity of capital, which will be a stationary variable in any general equilibrium model. Similarly the Hall-Jorgenson tax term is bounded and has tended to be mean-reverting.

To summarize, explicitly modelling the effects of adjustment costs tells us that the effect on investment of shocks to the cost of capital depends on the perceived persistence of the shocks. We have also shown that the persistence of the cost of capital varies substantially across equipment type, with the cost of capital for computers being dominated by the persistent decline in purchase prices. These results suggest using a disaggregated approach that allows different types of equipment to have different elasticities with respect to the cost of capital.

5 Econometric Modelling

5.1 Regressions

We estimated the capital stock adjustment formula, equation (20), for aggregate equipment as well as for computing and non-computing equipment. Because the proposed regressions contain nonstationary variables, we first addressed whether there is a cointegrating relationship. We ran the potential cointegrating regressions and applied Phillips-Ouliaris-Hansen tests for a unit root in the residuals. We could not reject the hypothesis that the error term has a unit root for any of the three categories. (This may be because our error term contains the biased technological change term $\hat{\eta}_t$, and it is possible that this term has a unit root.) These results indicate that the conventional approach in the "horserace" literature of differencing to avoid a spurious regression was probably well-founded.[11] We will follow this approach in estimating a differenced version of equation (20).[12]

[11] For completeness, we also estimated our regressions in levels; the important results of this section were unchanged.

[12] Note, though, that our approach of directly estimating the capital stock adjustment equation differs from the approach of the traditional models. These models applied repeated substitution of the lagged k_t term to transform the theoretical $ARMA$ equation into an $MA(\infty)$ equation, and then approximated this equation using an an $MA(n)$ regression. However, if the adjustment cost parameter, λ, is high (and empirical estimates suggest that it is), then terms omitted in this $MA(n)$ approximation will still have large coefficients. Since these terms are probably positively autocorrelated, we believe that this accounts for the poor autocorrelation properties of the traditional models.

The results are shown in Table 1. The aggregate results (column 1) are familiar from previous empirical investment papers. The estimated λ of 0.93 implies relatively slow adjustment. The sum of the coefficients on output is significantly positive and the sum of the coefficients on the cost of capital, though negative as expected, is quite small. The long-run elasticities are shown in the bottom part of the table. For the cost of capital, this elasticity is only -0.18.

The second column of Table 1 shows this regression for computing equipment. Limited data availability requires us to estimate over a smaller sample for computing equipment (1980-97), which leads to less tightly estimated coefficients.[13] Nonetheless, this column contains an important result: *The estimated long-run elasticity of the computer capital stock with respect to the cost of capital is -1.6, nearly 9 times the estimate from the aggregate model.* Column 3 reports the results for non-computing equipment; these are similar to the aggregate regression.

According to the model in the previous section, regressors with more persistent time series processes should have higher elasticities. Thus, part of the explanation for the larger cost-of-capital elasticity for computing equipment could be that the variance for the computer cost of capital is dominated by persistent shocks (falling computer prices). Columns 4-6 examine this hypothesis and provide confirmation. For both computing and non-computing equipment, the elasticities with respect to the more persistent components of the cost of capital (the relative price terms) are larger—in the case of computers, significantly so. Moreover, the long-run investment elasticity with respect to computer prices is also statistically significantly larger than the non-computer elasticity with respect to non-computer prices.[14]

In fact, by estimating the persistence properties of the various regressors we can calculate exactly *how much* higher the elasticities on persistent regressors should be. We estimated processes for price and non-price variables for both computing and non-computing equipment, using a stationary representation for the non-price variables, and imposing the

[13] We chose this starting data because the stock of computing equipment was very small before 1980. None of the results reported here are sensitive to the choice of sample.

[14] The results we have shown in this section are robust. Durbin's h statistics are low indicating that the regressions are free of residual autocorrelation. Specification changes (such as including a trend and adding extra lags) did not significantly alter any of our results. Furthermore, the regressions show no evidence of parameter instability in the 1990s.

assumption that the processes for the price variables are $I(1)$. Using these processes along with equations (17), (18), and (20), we find that the cross-equation restrictions implied by the model tell us that, for both computing and non-computing equipment, the conditional elasticity of the capital stock with respect to the non-price variables should equal about half the elasticity with respect to the price variables. A Wald test of these cross-equation restrictions reveals that they cannot be rejected. However, because of the relative imprecision of the estimates we are reluctant to place too much emphasis on these tests.

Our assumption that the relative price series are $I(1)$ also implies that the estimated long-run elasticities with respect to these variables should equal the elasticities of substitution for each type of capital. The implied elasticity of substitution for non-computing equipment is -0.33, in line with standard estimates from previous investment studies, although still perhaps surprisingly low. For computing equipment, the implied elasticity of substitution of -1.83 is extremely large. A possible interpretation of this result is that computer technologies are more easily substitutable for other factors.

5.2 Implications of Computer Price Measurement Error

One question about our large estimate of the elasticity of computer net investment with respect to its relative price is whether it could be affected by errors in the measurement of computer prices. The reasons to suspect that measurement error may be affecting this coefficient are twofold. First, the NIPA computer price index is a constant-quality series. This price is constructed from so-called "hedonic" price regressions, and there is certainly room for mis-specification and mis-measurement in these regressions. Second, like almost all NIPA expenditure categories, real investment in computing equipment is constructed by deflating the nominal expenditure series by the price index. Thus, any measurement error in the price index will affect both the right- and left-hand sides of our net investment regression.

While such measurement error may affect our regressions, we believe that consideration of this factor points to a price elasticity for computing equipment that is *larger* in magnitude than our estimate. This is because this type of measurement error biases the estimated long-run elasticity with respect to prices towards minus one and our estimate is -1.83. To illustrate this result, consider a simplified version of our theoretical investment equation, without dynamics or non-price cost-of-capital terms:

$$\Delta k_t = \alpha + \beta \Delta y_t - \gamma \Delta p_t + \epsilon_t$$

Suppose now that the NIPA price, p^*, is measured with error so that

$$\Delta p_t^* = \Delta p_t + u_t$$

The measured real net investment series is the nominal series divided by the measured price:

$$\begin{aligned}
\Delta k_t^* &= \Delta k_t + \Delta p_t - \Delta p_t^* \\
&= \alpha + \beta \Delta y_t - \gamma \Delta p_t + \epsilon_t + \Delta p_t - \Delta p_t^* \\
&= \alpha + \beta \Delta y_t - \gamma \Delta p_t^* + (1-\gamma)(\Delta p_t - \Delta p_t^*) + \epsilon_t \\
&= \alpha + \beta \Delta y_t - \gamma \Delta p_t^* - (1-\gamma) u_t + \epsilon_t
\end{aligned}$$

Note now that

$$Cov\left(-\Delta p_t^*, -(1-\gamma) u_t\right) = (1-\gamma)\sigma_u^2$$

Thus, the sign of the bias in the estimate of γ depends on the value of γ itself. If $\gamma < 1$, then the bias is positive, while if $\gamma > 1$ the bias is negative. Since our estimate of the coefficient on the relative price of computing equipment is greater than one in magnitude, this suggests that, if measurement error is a factor, then the true coefficient is greater in magnitude than our estimate.[15]

5.3 Out-of-Sample Forecasting

Our interpretation of the results in Table 1 is that they are broadly consistent with the theoretical approach outlined in the previous section. However, what of the fact that prompted this exploration, the investment boom of the 1990s? To test whether our two-equation procedure for predicting net investment helps to explain the recent behavior of the capital stock, we estimated our preferred equations for computing and non-computing equipment (Columns 5 and 6 of Table 1) through 1989:4. We then simulated them out

[15] In any case, we believe the evidence on NIPA price deflators suggests a sanguine interpretation of the measurement error problem. Recent research by Doms (1999) has shown that price declines measured from matched models (following the price of the same machine over time) are similar to the NIPA measures based on hedonic regressions.

of sample, taking the realized paths of output and the cost of capital as given, to obtain simulated capital stock series for computing and non-computing equipment.

Applying chain aggregation to our two simulated capital stock series, we obtained a simulated series for the aggregate capital stock. As shown in Figure 5, the two-equation system produces a series (the dotted line) that tracks the actual behavior of the equipment capital stock (the solid line) in the 1990s much better than the out-of-sample simulated series for the aggregate version of the same regression (the dashed line). The series generated by the aggregate regression, like the in-sample residuals from the aggregate net investment model in Figure 2, fall further and further behind observed capital stock growth as the 1990s proceed. In contrast, while the disaggregated system underpredicts actual capital stock growth somewhat for a number of periods from 1993 on, it moves back in line by the end of our sample (1997:4). The reason for the superior tracking performance of the disaggregated system is intuitive: This approach allows the massive decline in computing prices to feed through to capital accumulation far more than aggregate econometric regressions.

More important than the system's ability to track the aggregate capital stock, however, is its ability to explain the behavior of gross equipment investment. As the perpetual inventory depreciation rates for computing and non-computing are relatively stable over our sample, we can use a simple out-of-sample forecasting procedure for gross investment: We convert the disaggregated out-of-sample forecasts for capital stocks into forecasts for gross investment using the most recently observed depreciation rates. Applying this procedure to our system estimated through 1989:4 produces gross investment series for computing and non-computing investment. Aggregating these series, we obtain a good description of the recent behavior of aggregate equipment investment: Our simulated out-of-sample series for aggregate gross investment grows 6.9 percent per year over 1990-97, pretty close to the observed value of 7.5 percent. Moreover, as shown in Figure 6, our simulated series (the dotted line) captures the move to rapid investment growth in 1992 and the sustained high rate of growth thereafter. In contrast, an aggregate model—using the same specification and the 1989 aggregate depreciation rate—would have averaged about 3.1 percentage points too low over the period 1990-97 (the dashed line).

6 Conclusions

Boosted by exploding investment in computing equipment, the behavior of equipment investment in the U.S. in the 1990s has been unprecedented. Thus, it should not be too surprising that the traditional econometric models of investment, based as they are on historical correlations, have completely failed to explain the boom. We conclude that these developments provide three important lessons for macroeconomists:

- *Prices Matter*: Many previous studies have found limited roles for price variables, stressing the ability of an accelerator model to explain the cyclical behavior of investment. In contrast, we find an important role for equipment prices. Specifically, falling computer prices played a crucial role in the investment boom of the 1990s.

- *Depreciation Matters*: Most empirical studies have tended to ignore the role played by the replacement of depreciated capital. We have shown that an increasing depreciation rate was of first-order importance in the extraordinary behavior of equipment investment in the 1990s. Moreover, we have pointed to an important issue in the measurement of depreciation rates: Methodological changes to the NIPAs have made the standard measure of the average depreciation rate based on aggregate data invalid.

- *Aggregation Matters*: Depreciation rates vary widely across different types of equipment. Also, a model with rational expectations and adjustment costs tells us that the effects of cost of capital shocks will not be uniform across all types of equipment. We show that a two-equation system for net and gross investment in computing and non-computing equipment, estimated through 1989, is capable of explaining the magnitude and pattern of the U.S. equipment investment boom of the 1990s, while aggregate models completely fail.

Put simply, our explanation of equipment investment in the 1990s is that declining computer prices had a very large effect in boosting the accumulation of computer capital. Consequently, this led to even greater rates of replacement investment. Ultimately, of course, the true test of any model is its ability to forecast future developments. We hope that the future does not turn out to be as unkind to our empirical approach as the 1990s proved to be to the traditional econometric models.

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Appendices

A The Data

Our dataset consists of quarterly series over 1950:1-1997:4 for real output, as well as real investment, real capital stock, and the cost of capital for total equipment, computing equipment, and non-computing equipment. Our output series is real 1992 dollar output of the private business sector, which is defined as GDP minus output from government and non-profit institutions and the imputed income from owner-occupied housing.

Our series on real investment for total equipment is private nonresidential producers' durable equipment expenditures from National Income and Product Accounts (NIPA) Table 5.5. The data for real computer expenditures is the Computers and Peripherals series from the same source. For real capital stock series for total equipment and computing equipment, we started with the annual NIPA capital stock data, which are available through 1997 and published in Department of Commerce (1998). These annual data, which represent year-end stocks, were then converted to quarterly series using an interpolation routine that sets the growth rate for each quarter according to its share in the annual total for investment expenditures.

Our series on real equipment investment and real capital stock excluding computing equipment were not created by subtracting the real series for computing equipment from the real aggregates. The lack of additivity of the chain-aggregation formula means that this is an incorrect calculation. Rather, in theory, we need to construct a new aggregate from the 26 disaggregated non-computing equipment categories. In practice, a "chain-subtraction" procedure which applies equation (9) to aggregate equipment and the negative for computer investment works just as well and does not require data on 26 investment series.

The cost of capital is measured using the Hall-Jorgenson rental rate formula:

$$C_t = P_t \left(R_t + \delta - \frac{\dot{P}_t}{P_t} \right) \left(\frac{1 - ITC - \tau * DEP}{1 - \tau} \right)$$

where P_t is the price of capital relative to the price of output, R_t is the real interest rate, ITC is the investment tax credit, DEP is the present value of depreciation allowances, and τ is the marginal corporate income tax rate. The relative price series, P_t, are defined relative to the deflator for private business output. The "capital gains" term is implemented as a three-year moving average of the percentage change in P_t. To construct the real interest rate, R_t, we subtracted expected inflation – proxied by the average inflation rate of the private business output deflator over the previous five years - from the nominal rate on Baa corporate bonds. We then added a constant "risk premium" that normalized this required rate of return so that its average equalled the average rate of return on physical capital in our sample (6.8 percent), where this is measured as the ratio of nominal capital income to the nominal capital stock. The tax term was constructed using data on investment tax credits and service lives (used in the calculation of depreciation allowances) from Gravelle (1994). We used $\delta = 0.31$ for computing equipment, $\delta = 0.13$ for non-computing equipment, and a nominal-capital-stock weighted average of these two rates for aggregate PDE. We use a nominal capital stock weighted average because of the problems with aggregate perpetual inventory depreciation rates discussed in Section 3.

B Depreciation Rates with Chained Aggregates

What will the calculated aggregate depreciation rate from equation (8) look like with chain-aggregated data? To keep the analysis transparent, we will look at a simple case. The Fisher chain formula is somewhat cumbersome, so instead we will use the Tornqvist aggregation formula. This procedure weights the growth rate of each category according to its share in the nominal aggregate, and produces aggregates with almost identical properties to the Fisher procedure. We will also make the following assumptions. Both types of capital depreciate at the same rate δ; the price of type-A capital falls at rate γ relative to the price of type-B capital and output, which are both normalized to equal one. Finally, firms produce with a Cobb-Douglas production function ($Q_t = A_t \left(K_t^A\right)^\alpha \left(K_t^B\right)^{1-\alpha}$) and there are no adjustment costs. Now, assuming no taxes, the cost of capital for type A simplifies to $P^A(r + \delta + \gamma)$. The cost of capital for type B is $(r + \delta)$. Given our assumptions, firms accumulate capital according to the first-order conditions:

$$K_t^A = \frac{\alpha Q_t}{P_t^A(r + \delta + \gamma)}$$

$$K_t^B = \frac{(1 - \alpha) Q_t}{(r + \delta)}$$

These conditions imply capital stock growth rates $g^A = g^Q + \gamma$ and $g^B = g^Q$. Using the Tornqvist formula, the growth rate for the chain-aggregated capital stock is

$$g^{CW} = \theta(g^Q + \gamma) + (1 - \theta)g^Q = g^Q + \alpha\gamma$$

where θ is the share of capital of type A in the aggregate nominal capital stock. Note that these nominal stocks are defined as the "replacement value" of the capital stock and are obtained by reflating the real capital stock for each category by the current-period price of new capital. Now consider the behavior of a chain-aggregate for real investment. Re-arranging the expressions for the growth rate of the capital stock we get:

$$\frac{I_t^A}{K_{t-1}^A} = g^Q + \gamma + \delta$$

$$\frac{I_t^B}{K_{t-1}^B} = g^Q + \delta$$

Thus, for each type of capital, the ratio of real investment to the real capital stock is a constant. So, real investment for capital of types A and B also grow at rate g^A and g^B.

To calculate the growth rate of the chain aggregate, we need nominal shares of investment:

$$\begin{aligned}
\frac{P_t^A I_t^A}{P_t^B I_t^B} &= \left(\frac{I_t^A}{K_{t-1}^A}\right)\left(\frac{K_{t-1}^B}{I_t^B}\right)\left(\frac{K_t^B}{K_{t-1}^B}\right)\left(\frac{K_{t-1}^A}{K_t^A}\right)\left(\frac{P_t^A K_t^A}{P_t^B K_t^B}\right) \\
&= \left(\frac{g^Q + \delta + \gamma}{g^Q + \delta}\right)\left(\frac{g^Q + 1}{g^Q + 1 + \gamma}\right)\left(\frac{P_t^A K_t^A}{P_t^B K_t^B}\right) \\
&> \left(\frac{P_t^A K_t^A}{P_t^B K_t^B}\right)
\end{aligned}$$

The share of capital of type A in nominal *investment* is larger than its share in the nominal *capital stock*. The reason for this is intuitive. The real capital stock of type A is growing faster than the real stock of type B. This means that, measured in today's dollars at replacement cost, there is more investment relative to the capital stock for type A than there is for type B; as a result the nominal share of investment for type A is higher. Since real investment of type A grows at rate $g_Q + \gamma$ while real investment of type B grows at rate g_Q, the growth rate of the Tornqvist chain aggregate for real investment places more weight on the faster growing category than does the corresponding growth rate for the aggregate capital stock. Hence, the chain aggregate for investment will always grow faster than the chain aggregate for the capital stock. This example, in which relative price shifts cause the fast growing category to have a larger share in nominal investment than in the nominal capital stock, lines up precisely with reality: Computers currently have a much larger share in nominal equipment investment (14 percent in 1997) than in the nominal equipment capital stock (5 percent in 1997).

Now, suppose we solve for the aggregate depreciation rate from the chain-aggregates for investment and the capital stock:

$$\delta_t^{CW} = \frac{I_t^{CW}}{K_{t-1}^{CW}} - g_t^{CW}$$

Then this value will equal δ only in the base year. Since I^{CW} grows faster than K^{CW} in each period, this "depreciation rate" gets larger each period. More generally, if we allowed the two types of capital to have varying depreciation rates, the depreciation rate estimated from this equation would only equal a weighted average of the underlying depreciation rates in the base year, as we move forward from the base year this measure would eventually be higher than each of the underlying depreciation rates.

C Omitted Proof

Proof that $\mu(1)(1-\theta\lambda) = 1$ when $\pi(1) = 1$:

Inserting the expression for $\mu(1)$ we need

$$(1-\theta\lambda)\left[1+\sum_{k=1}^{m-1}\left(\sum_{r=k+1}^{m}(\theta\lambda)^{r-k}\pi_r\right)\right] = 1-\pi(\theta\lambda)$$

Re-arranging the left-hand-side of this equation we get

$$(1-\theta\lambda)\left[1+\sum_{k=1}^{m-1}\left(\sum_{r=k+1}^{m}(\theta\lambda)^{r-k}\pi_r\right)\right] = (1-\theta\lambda)\left[1+\sum_{k=1}^{m-1}\left(\sum_{r=k+1}^{m}\pi_r\right)(\theta\lambda)^k\right]$$

Now use $\pi(1) = 1$:

$$(1-\theta\lambda)\left[1+\sum_{k=1}^{m-1}\left(\sum_{r=k+1}^{m}(\theta\lambda)^{r-k}\pi_r\right)\right] = (1-\theta\lambda)\left[1+\sum_{k=1}^{m-1}\left(1-\sum_{r=1}^{k}\pi_r\right)(\theta\lambda)^k\right]$$

Expanding this expression we get

$$\begin{aligned}
& 1+(1-\pi_1)(\theta\lambda)+(1-\pi_1-\pi_2)(\theta\lambda)^2+......+(1-\pi_1-\pi_2-..-\pi_{n-1})(\theta\lambda)^{m-1} \\
& -\theta\lambda-(1-\pi_1)(\theta\lambda)^2-(1-\pi_1-\pi_2)(\theta\lambda)^3-..........-.(1-\pi_1-\pi_2-..-\pi_{n-1})(\theta\lambda)^m \\
= \quad & 1-\pi_1(\theta\lambda)-\pi_2(\theta\lambda)^2-...-\pi_n(\theta\lambda)^n \\
= \quad & 1-\pi(\theta\lambda)
\end{aligned}$$

as required.

Table 1

Capital Stock Growth Regressions

(Standard errors in parentheses)

	Total (50-97)	Computers (80-97)	Excluding Computers (50-97)	Total (50-97)	Computers (80-97)	Excluding Computers (50-97)
λ	0.93	0.90	0.92	0.94	0.86	0.93
	(.02)	(.04)	(.02)	(.02)	(.05)	(.02)
Sum of the Coefficients on:						
Output	0.10	0.18	0.11	0.10	0.21	0.10
	(.01)	(.13)	(.02)	(.01)	(.10)	(.02)
Cost of capital	-0.02	-0.17	-0.01			
	(.01)	(.09)	(.005)			
Relative prices				-0.03	-0.26	-0.03
				(.01)	(.10)	(.02)
Cost of capital without prices				-0.01	-0.08	-0.01
				(.005)	(.08)	(.006)
Long-run Elasticities:						
Output	1.49	1.75	1.33	1.46	1.44	1.33
	(.36)	(1.41)	(.31)	(0.37)	(0.84)	(.32)
Cost of capital	-0.18	-1.59	-0.13			
	(.09)	(.75)	(.083)			
Relative price of capital				-0.45	-1.83	-0.33
				(.26)	(.45)	(.31)
Cost of capital without relative price				-0.18	-0.53	-0.14
				(.11)	(.50)	(.10)

Figure 1
The Demise of the Traditional Investment Regression

Investment relative to the Capital Stock

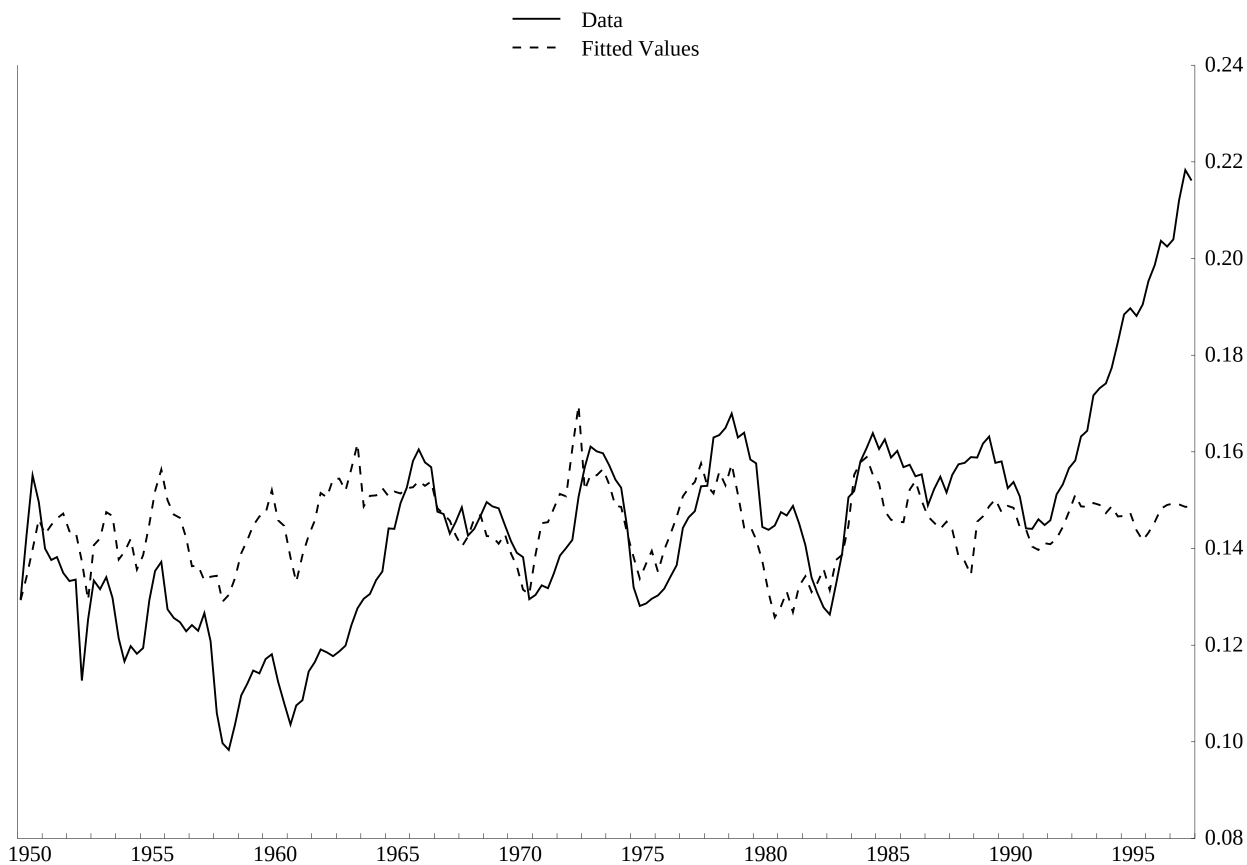


Figure 2
Perpetual Inventory Depreciation Rates for Aggregate Equipment

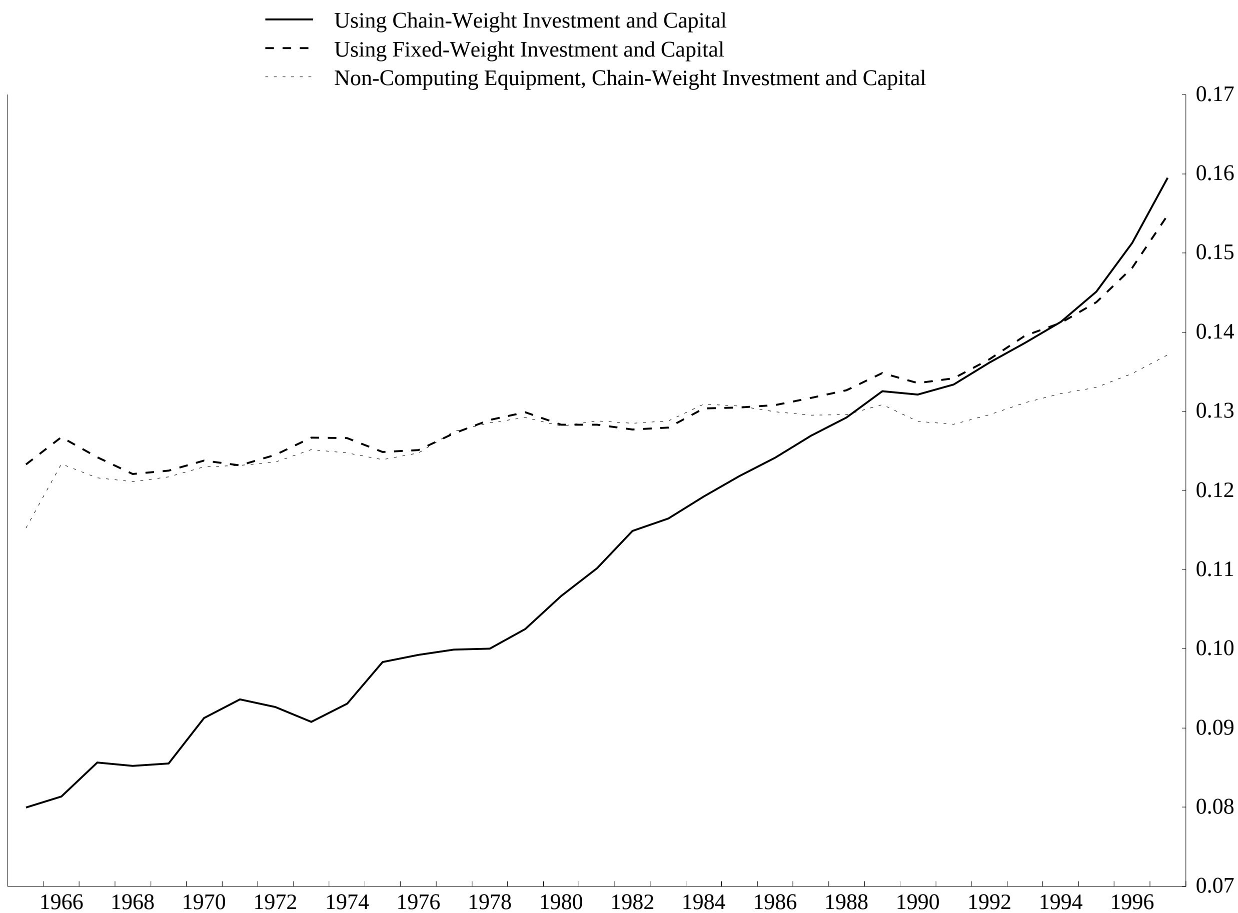


Figure 3

Residuals from Gross and Net Aggregate Equipment Investment Regressions

Errors Expressed As Percentages of the Capital Stock

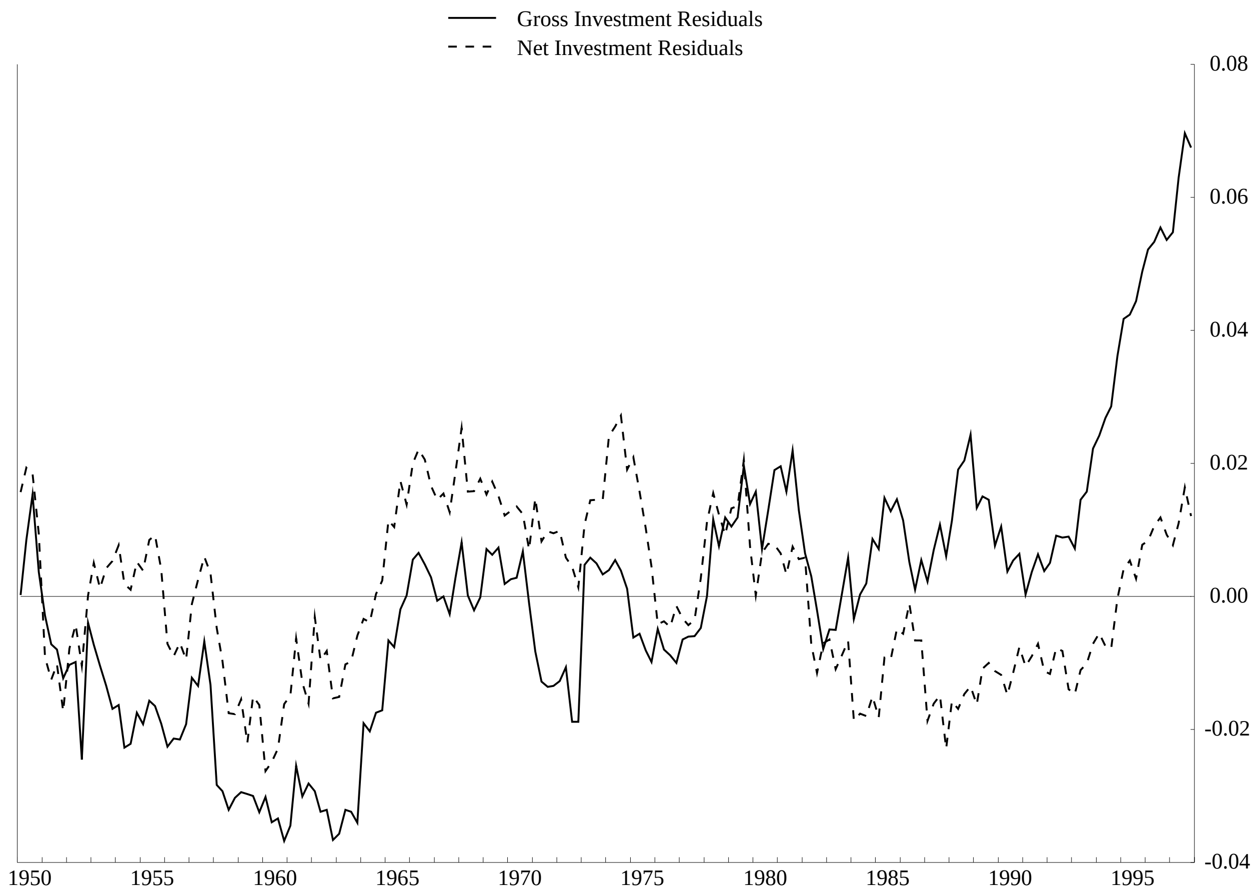


Figure 4
The Cost of Capital
In Logs

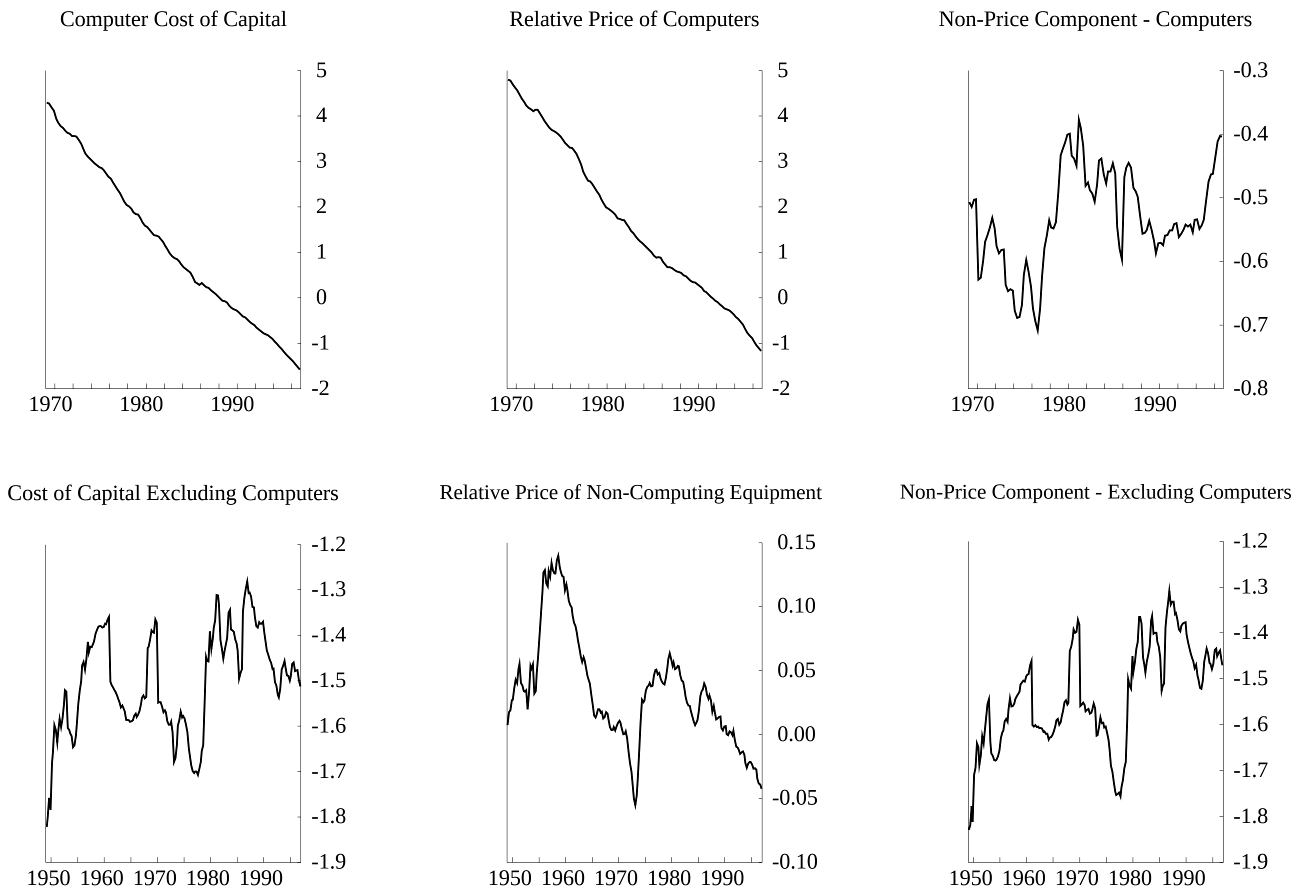


Figure 5
Out-of-Sample (Post-1989) Forecasts of Growth in the Stock of Equipment
Annualized Growth Rates

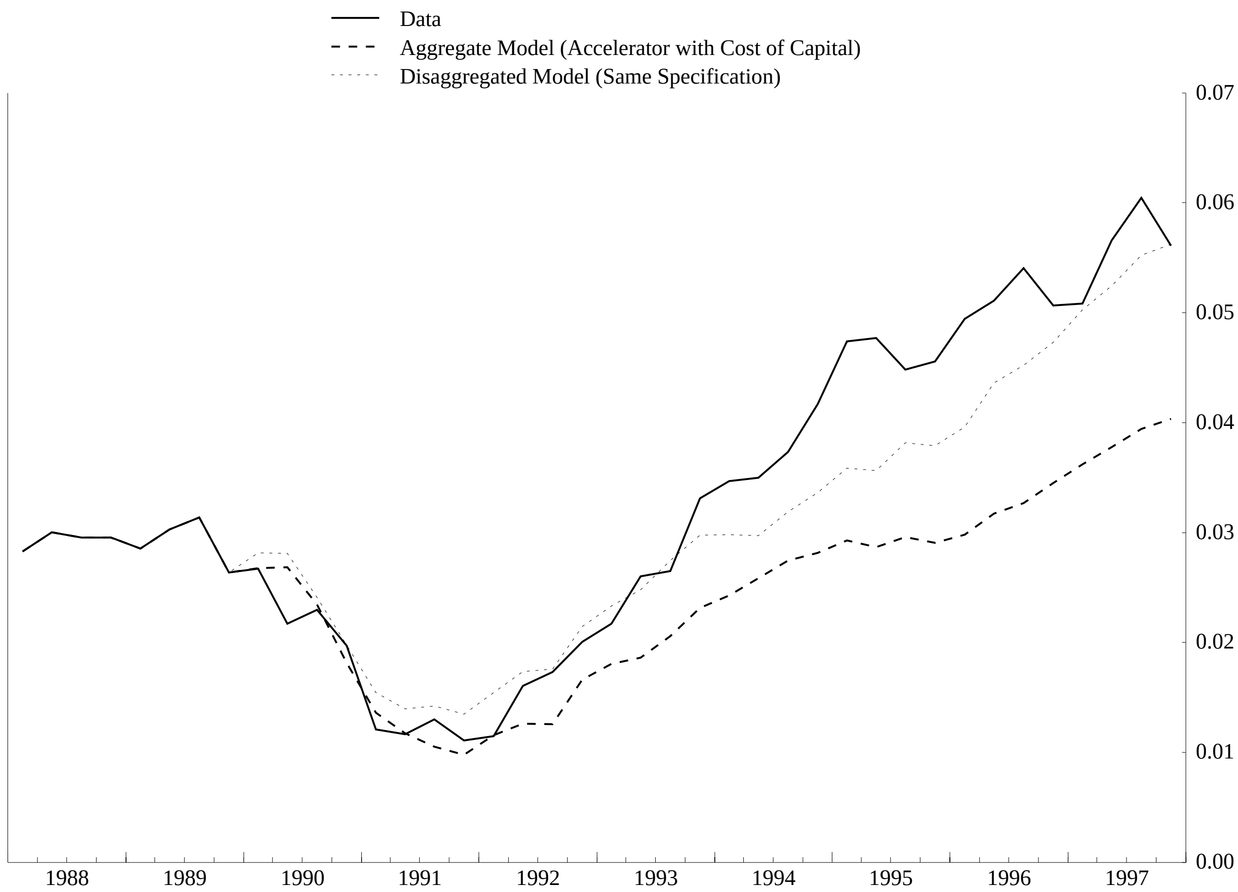
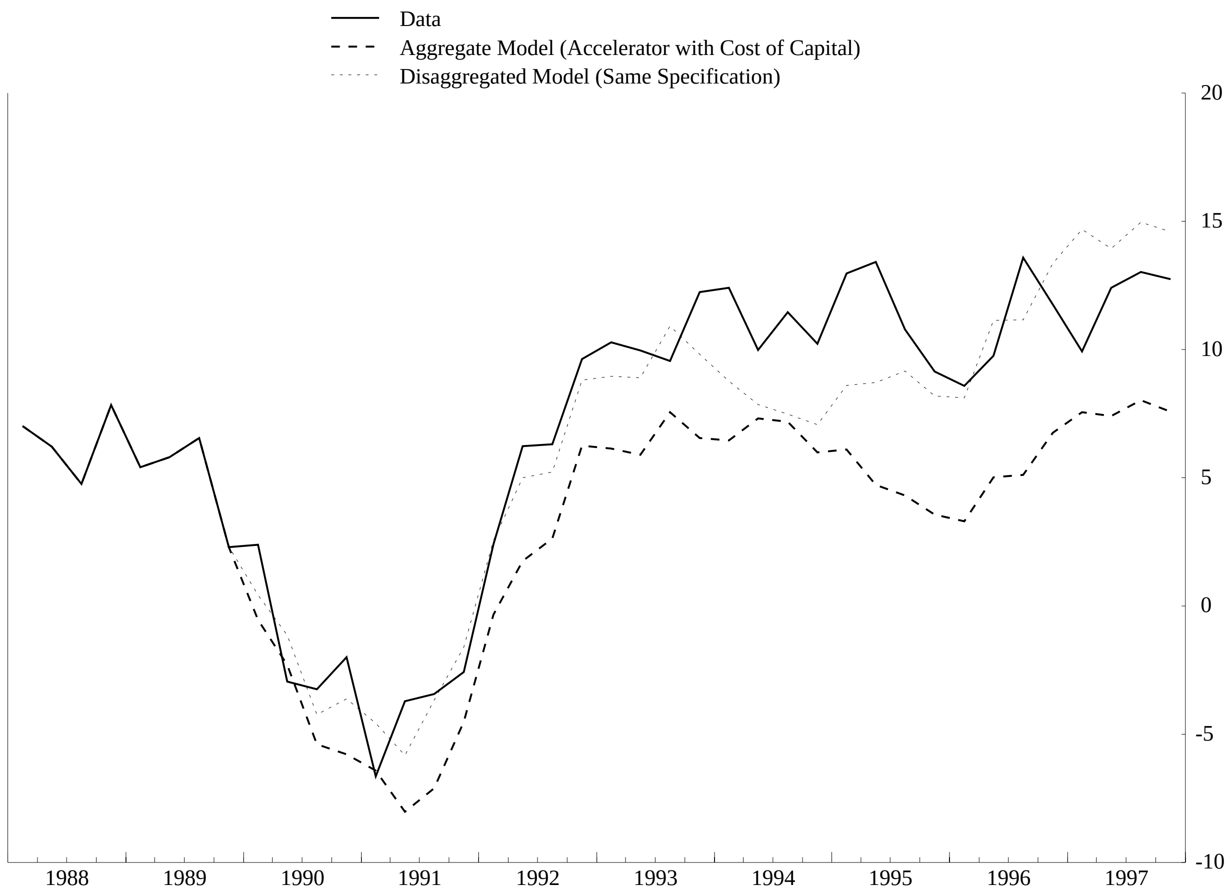


Figure 6
Out-of-Sample (Post-1989) Forecasts of Growth in Real Equipment Investment
Year-over-Year Percentage Changes



Estimates of the volume of capital services

Prabhat Vaze
Office for National Statistics

This article presents estimates of the volume of capital services for the United Kingdom as a whole as well as by industry. This experimental measure complements the wealth measures presented in the National Accounts and builds on the work done to improve these measures. The volume index of capital services weights together the growth in the net stock of assets using shares that reflect the relative productivity of the different assets that make up the capital stock. The article describes the method used to do this and explores the impact of treating ICT goods separately. Data related to current work and results are available at the National Statistics website: http://www.statistics.gov.uk.

Introduction

The Office for National Statistics (ONS) recently published improved estimates of the wealth measures of the capital stock and associated series such as capital consumption. Apart from chain-linking the volume estimates of capital stock, ONS's work provides greater industrial detail in the measures and also includes a long time-series of capital formation by industry and broad asset group (see Vaze, Hill, *et al.*, 2003). These data can be used when calculating new measures of the capital stock, including those that take account of different productivity of different asset types.

How the capital stock impacts on growth has become a topic of much interest. The link between productivity growth and investment has been discussed with particular reference to the recent large investments in assets related to the new economy, such as computers. While discussion and analysis continues, there has been a parallel debate about the measurement of capital. ONS wealth measures of capital value the replacement cost of the stock of capital as if new (gross capital stock) or taking account of the loss of value due to depreciation (net capital stock). However while these measures are useful for productivity work, there is a growing body of work proposing alternative measures that quantify the flow of input from the capital stock into production. Using these measures for capital input, analysts have 'accounted' for growth quantifying that part attributable to the input of capital. For the United Kingdom, recent examples of research in this area have been Oulton (2001) and O'Mahony and de Boer (2002).

Defining and measuring the contribution of capital to production has been a controversial issue, but a measure of international agreement has been reached in recent years. The issues involved and ways forward have been detailed in a recent manual by the Organisation of Economic Co-operation and Development (OECD, 2001). One suggested development is to disaggregate capital formation into a number of asset types. Research has indicated the sensitivity of UK capital stock measures to the separate treatment of assets with a short life-length, such as computers (Oulton and Srinivasan, 2003). Currently, the ONS quarterly-published investment series separates the main tangible assets as new building work, plant and machinery, and vehicles in both current and chain-linked volume measures. More detail in terms of assets is available in current prices in ONS supply-use tables and in capital formation surveys.

This article gives the results of the ONS work on an index of capital services to provide a measure of capital input into production and to complement the current wealth measures. The raw data for this is identical to ONS net and gross capital stock measures: long time-series of capital formation by asset, deflators by asset and defined assumptions about the asset decay and retirement pattern. The model employed owes much to work undertaken at the Bank of England, using the methodology described in Oulton and Srinivasan (2003).

Measuring capital input

The methodology to calculate a Volume Index of Capital Services (VICS) is described by Oulton and Srinivasan (2003) and the OECD Capital Stock manual also provides an invaluable resource (OECD, 2001). In summary, the stages are:

- aggregating the history of each asset's capital formation by industry over time with the different vintages of assets added together in a manner reflecting decay;
- pricing asset's services using the estimated rental for each asset;
- aggregating across assets, weighting the stocks in an index reflecting their input into production.

Generally, there is a decline in the productive potential of an asset as it decays over time. So it is better to add together the assets using weights that reflect this decay with newer assets having a higher weight. The decay of an asset over time is approximated by its age-efficiency profile. A function such as straight-line decay has sometimes been used, but a 'smooth' function used here is the infinite geometric decay function. This has some elegant mathematical properties, which greatly simplify the analysis of the capital services – the box indicates two of the implications. But most of the decay occurs at the start of the asset's life, which can be questioned.

Given a decay function, it is possible to convert time-series data about the volume of purchases of assets into a stock measure. The stock measure reflects the sum of the assets, weighted together to reflect the different efficiencies of the various vintages of the assets. For example, if the selected age-efficiency profile is geometric with 10 per cent decay per annum, then 90 per cent of the asset will remain after the first year, 81 per cent in two years and so on. In calculating decay rates, we use the average life-length in years assumed for each asset and each vintage in the ONS national accounts stock models, converted into a rate using a method explained in the annex.

A second area is the pricing of an asset's services over time, given by the rental. Rentals are the payments made for the year's service of a capital good. An efficient firm would equate the marginal returns of the services of an asset in a period to the rental of the asset. In some circumstances, there is a rental market and the rentals may be directly observed. For example, this is the case in office space and some machinery. However, for many goods, the rentals are not observed and a model is used to impute the asset's rental. The basis for this is asking the question what would the owner of an asset expect to be paid for a year's use of the asset?

There are three costs associated with renting an asset and an adjustment reflecting the taxes and subsidies that accompany an investment. Firstly, over the year, the asset loses value due to decay or ageing and some part of the rental will reflect this. To model the value of this component, the decay rate used in calculating the stock measure is used. A second part of the rental is due to changes in the price of a new asset. These are the holding gains or losses reflecting the value change of an asset in the year due to aspects other than ageing, such as capital gains in property. A final cost is the cost of capital.

Consistency issue 1:
Linking price of a capital asset with the decay of capital

The decay model geometric, light bulb, straight-line and so on – allows us to model the future volume of an asset's capital services over its lifetime. That model allows the analyst to predict the future productive behaviour of the asset. We can link that with the price of the capital services (the rentals) to calculate a series for the future values of capital services. The present value of the future stream of the asset's capital services could be calculated, taking the decay of the asset into account. This present value would equal a measure of the net stock of the asset. Changes in the present value as assets age would be depreciation. In some statistical systems, such as that in Australia, the net stock measures and depreciation are consistent with the decay function that is used in asset modelling. Here, the net stock used in the volume index of capital services is consistent with the rentals measure, but the measures used in the UK wealth measures, published in the national accounts, uses straight-line depreciation.

Consistency issue 2:
Rates of return and operating surplus

The sum of the value of capital services is a measure of the operating surplus. If the rentals are calculated assuming a rate of return on capital – such as a government bond rate – it is unlikely that the total value of the capital services will equal the observed operating surplus. However, it is possible to calculate an ex post rate of return such that the rate of return to exhaust the operating surplus in the economy. The rate of return is then calculated endogenously. Here dwellings are not modelled as part of the productive capital stock and the part of operating surplus attributable to dwellings has been deducted from the total gross UK operating surplus, as measured by owner-occupier imputed rents and the depreciation of the stock of dwellings. It would be possible to calculate industry-specific rates of return using industry operating surpluses. But this has not been done in the present analysis and instead one rate is assumed across all industries.

The owner of the asset could have sold the asset and put the monies into an alternative interest-bearing financial asset. The rental ought to compensate the owner for this opportunity cost. The sum of these three costs is adjusted for taxes or subsidies available on investments.

Having calculated the stock of an asset and the rental for that asset, it is now possible to multiply the rentals by the stock of capital to give the value of capital services provided by an asset over a year. This is done for each of the assets and then added together. The sum of the value of capital services is a measure of the total value added by capital goods in the production process. It would be a current price gross measure rather than net, as it would include the depreciation of the capital stock and could be compared with the gross operating surplus given in the production accounts of the national accounts.

The volume of capital services is the volume or real measure of these capital services and is calculated by an index aggregating the growth in the stock of individual asset (volumes) using appropriate weights. The index used here – the chain-linked Laspeyres – has not been found to give significantly different results to the Tornqvist (for example, used in Oulton (2001)) and is consistent with the current UK chain-linked macroeconomic aggregates. The weights used in the index are the shares of the assets in the value of capital services in the previous year. Under an assumption of profit maximisation and market competitiveness, it can be shown that these shares approximate the elasticity of output to the volume of capital services inputting into the production process.

ONS work on capital services has some particular features. Firstly, the model estimates stocks and rentals at a very disaggregated industrial and asset level. For each asset and industry, a long time-series of investment is used to derive stocks and these are weighted together using shares based on rentals modelled for each asset. Thirty-five industries and between one and six assets for each industry have been modelled. This very disaggregated modelling is also a feature of O'Mahony and de Boer (2002), which provides some comparison.

The ONS model allows the depreciation rate of the assets to vary over vintage, that is, the life-length of an asset will vary depending on the year of purchase. Although changes across time are infrequent, a general observation is that life-lengths of assets have lessened over time. This reflects both shortening in the life-length of assets due to reviews of the assumptions made by ONS and also a shift to short-lived assets.

Investment in computers

The heightened interest in measuring capital has particularly focused on the role of investment in information and communication technology (ICT) goods and services in recent years. The level and growth in ICT investment has to be seen in the light of two key features of ICT capital. Firstly, ICT assets have shorter life-lengths than the other main asset types. (For example, in the United Kingdom, the vehicles asset type has one of the shortest assumed life-lengths, but computers and software typically last less than half this length of time at about five years.) A second feature common to ICT goods and services is large annual falls in prices due largely to improvements in quality.

The shorter life-lengths and the rapid falls in prices have led to analysts separating out the ICT assets in their capital stock models (Oulton, 2001; O'Mahony and de Boer, 2002; OECD, 2001). ONS is also moving in this direction – ONS introduced into its wealth measures of the capital stock a new category of numerically controlled machinery in the 1990s and computers are modelled separately in these measures. In modelling capital stock, long time-series of investment data by asset is needed. Estimates of computer investment have been improved as part of the current work. This has been achieved through using the detailed product breakdown available in the investment tables included in the supply-use tables, produced by ONS. The supply-use tables accompanying *Blue Book 2002* provide annual data from 1992 onwards giving the 123-product breakdown and 35 industries.

For the years before 1992, there are a number of measurement issues surrounding the long time-series history of capital formation in computers that is needed. Suitable Input-Output data is available from 1979. But different industrial classification systems were used over the period and the tables were produced infrequently prior to 1989. Information to convert between different vintages of the standard industrial classification (SIC) was used to make a consistent SIC92 series for computers. Where input-output tables are not available, the share of computers in plant and machinery was interpolated and this was then applied to the annual data available in national accounts on plant and machinery investment.

Computer prices have been falling much faster than other assets. This partly reflects the improvements in quality measured using an option-cost method since the late 1990s and a hedonic regression since the start of 2003 (Ball and Allen, 2003). To measure the productive capital stock, some account of these prices falls must be taken and to allow this the current work has separately deflated the investment in computers using the price index (ONS code PQEK). The current work has also then removed this PPI from the industry-specific deflators for plant and machinery having a positive effect on asset price growth. This has been done for all years after 1995.

Technology assets such as computers largely motivate the need for measures of the capital stock that take account of the different productivity of assets. The rationale can be seen by comparing computers with a long-lived asset such as buildings. A similar value of investment in the two assets would both provide different patterns of services to a business for a period beyond the year the investment takes place. In the case of computers, the life-length being shorter and falling prices both mean that current price investment would need higher returns for computers and the VICS recognises this by attributing a higher productivity for computers.

A second significant impact of the price falls seen in computers is observed in the growth in the stock of computers. As old vintages of computers are being retired, new, more powerful computers are replacing them. The growth in the net stock for this asset reflects the increasing current price expenditures on computers. However, even in years where the current price growth is modest or falling, an upward impact on the constant price net stock of computers occurs due to the replacement of old computers with new more powerful ones.

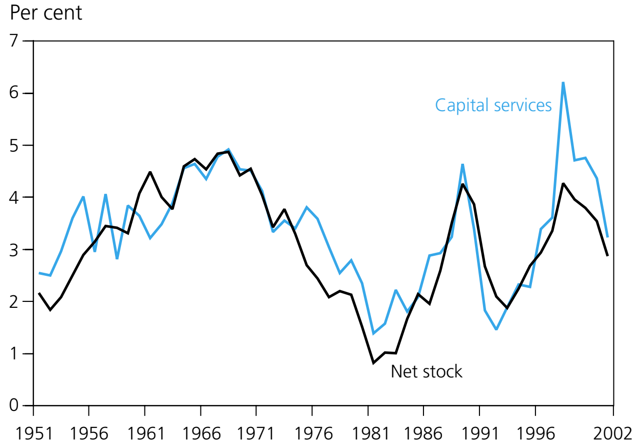
Analysing capital services

Table 1 presents a series for the volume index of capital services for UK whole economy and by industry. The results currently cover the period up to 2002. Figure 1 indicates the growth rates of volume index of capital services for the whole economy during the period 1950–2002. The early periods show strong and sustained growth in the input of capital goods into production, particularly in the second half of the 1960s. However, this early period suffers from one notable measurement issue – quantifying the one-off loss of capital stock due to the Second World War (Dean, 1964, provided the official estimates of this, which is used here). The period of the 1970s however saw more modest growth reaching a low in 1982, a period when investment in many industries was below replacement levels. The 1980s and 1990s have similar patterns in both decades beginning with low growth rates, but the second half of each decade seeing strong investment and so strong growth rates for the VICS.

Also included on the figure is the change in the net capital stock excluding dwellings. The close fit is to be expected given both measures weight together the changes in the net stock. Net stock measures are underpinned by the same datasets, namely the long time-series investment by asset, price indices and assumed life-lengths. However, some differences are expected due to the different construction of the indices, particularly the weighting of asset growth by their profit shares in the VICS, rather than in asset value terms in the net stock. The most pronounced differences occur when this effect would be high – primarily, in the late 1990s. During this period, investment in computers was growing fast and the price of computers falling markedly. The latter would make the share of computers in the index high. This combines with growth in computer investment to raise the VICS above the net stock measure. In 1998, the VICS grew by 6.2 per cent, higher than the net stock growth of 4.3 per cent. The slowdown of investment in computers in the first years of this decade would reverse the growth in VICS as the weight of computers would remain high, but this being associated with slower growth in the net stock.

The VICS model endogenously generates the rate of return that exhausts the operating surplus. This methodology is noted in the box and is identical to that used in Oulton (2001). Comparisons between the current estimates of rate of return with Oulton (2001) indicate that there is little difference even though there has been a disaggregation into more industries and other differences in the two models. The estimated rate of return, which is a nominal measure, is then used in calculation of the rentals. It is common in such modelling that the estimated rentals sometimes are

Figure 1
Annual growth in measure of capital stock, 1950–2002



negative, and some assumption has to be made to remove such anomalies. In the current work, where negative rentals are estimated, the previous positive rental is used instead.

Capital services by industry

Figures 2 and 3 give the volume index of capital services by industry, indicating the average chained (Laspeyres) volume of capital services input for each industry. Also, for most industries, the minimum and maximum growth rates observed are also given, though for some industries these have been suppressed. Table 1 gives the time series of VICS for whole economy and by industry.

In the manufacturing industries displayed in Figure 2, over the period all have average growth below five per cent. There is some similarity in the growth rates observed in the output of industries and in the VICS. Industries that have seen a marked decline over the period – such as Basic metal products or Textiles – have also seen low growth in the volume of capital services used. Industries related to the oil and chemicals sector have shown stronger growth, as have those industries associated to information technologies, Electrical and optical equipment and Pulp, paper, printing and publishing, for example.

Figure 3 shows the contrast with services industry: all but six of the industries have VICS growth rates above five per cent. The picture is of generally high growth in capital services, reflecting strong investment over the period. All the major service industries where output is primarily provided by private businesses – retail, finance and other business services being the largest – indicate strong growth in capital services during the period. Table 1 shows when the maximum and minimum growth rates from Figures 2 and 3 occur. It can be noted that the strong annual rises in the VICS in the late 1980s were associated with strong growth in finance. The late 1990s growth in the VICS has a strong contribution from this industry. However, it is the growth in the post and telecommunications industry that is most pronounced with the VICS for this industry reaching 18 per cent growth in 1998.

Figure 2
VICS growth rates by production industry, 1950–2002

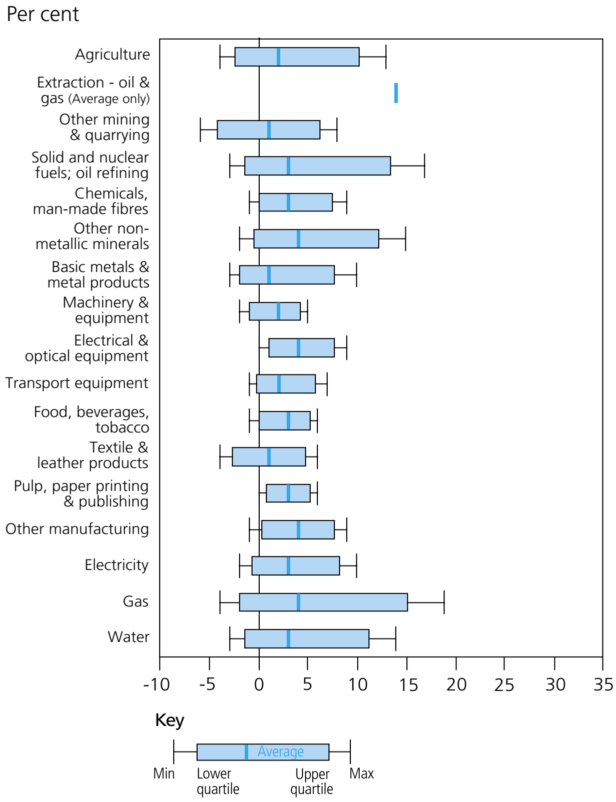
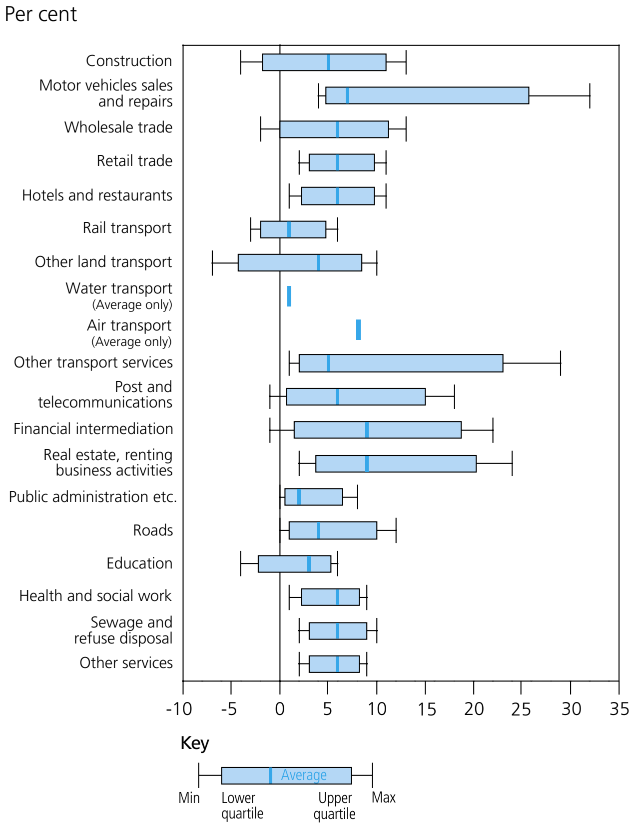


Figure 3
VICS growth rates by services industry, 1950–2002



Profit shares by asset

The weight of each asset in the volume index of capital services is the share of the total gross operating surplus attributable to each asset. These profit shares reflect a business's need to cover the decay of the asset (higher for short-lived assets such as computers) and to make a rate of return on finance tied up in the asset stock. Also, the business may gain from capital gains reducing the need for profits (as is sometimes the case in buildings), but may also see asset value lowered by factors other than depreciation.

Figure 4 and Table 2 indicate a change in the composition of the profit shares. Broadly over the period, the weight of buildings has declined as the share of plant has increased. The increase in the importance of plant and machinery – in its broadest senses including ICT – has motivated the breakdown of this asset into more categories. Since the mid-1980s the profit share attributable to computers has risen to approximately six per cent. The asset 'intangibles' is dominated by own-account software and contributes approximately 3 per cent of profits. However, this underestimates the importance of software as plant and machinery includes purchased software. The share of profits attributable to buildings has declined over the period, though it can be seen that the first half of the 1990s saw a steep rise in the share of this asset in profits. This reflects the positive impact on rentals of the modest growth and – in some years – decline in the price of buildings.

Computer investment and capital services

The investment in computers and other ICT assets observed in the 1990s motivates the modelling of the productive capital stock. To indicate the importance of this in the current analysis, the VICS model was run aggregating computers with plant and machinery. The combined asset of plant and machinery and computers was modelled with a set of life-lengths and price indices for each industry, which weighted together the measures for the two assets appropriately. The effect was to create an asset with a life-length between that of plant and machinery and computers. The price index of computers used in the VICS is identical to that for the rest of plant and machinery for the period to 1995. However, after this, a separate price measure is used for computers, which falls faster than the prices seen in plant without computers. These two indices were combined for each industry to give a plant deflator including computers.

Figure 5 compares the VICS with one calculated without computers as a separate asset (VICS ex computers). The period when investment in computers has been separated from other plant and machinery is 1980 onwards and the period until 1995 reflects the effect of having different life-lengths but the same price index for both assets. The indices track each other quite closely. However, after 1995, with a separate price index for computers being used to deflate the current price investment in this asset, the VICS diverges from the VICS modelling computers with plant. The third section gives some of the reasons why the VICS is greater if

Figure 4
Profit shares by asset, 1950–2002

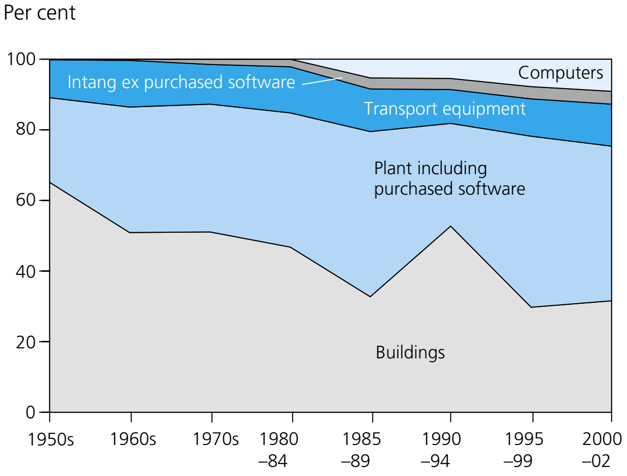
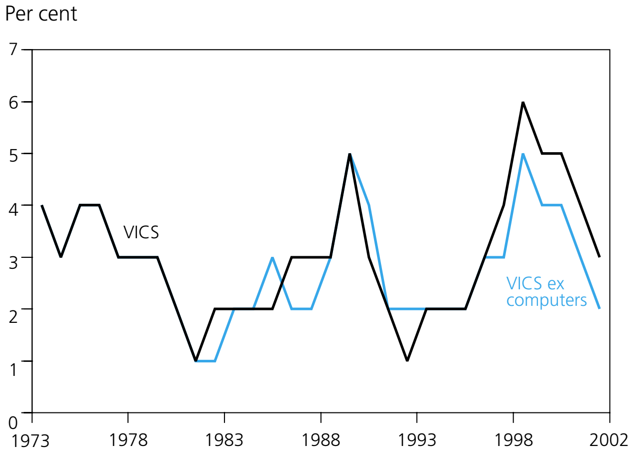


Figure 5
Impact of computers on VICS annual growth, 1972–2002



computers are modelled separately and so it is unsurprising that this proves the case.

Conclusions

This article accompanies the release of a volume index of capital services for the United Kingdom. The release is experimental and builds on continuing work improving ONS measures of the capital stock. This article has described some of the steps in measuring capital services and comments are welcome. While the VICS measure is related to the existing ONS capital stock measures, it should be noted that they are not a replacement. There is on-going work on the wealth measures of capital stock – net stock and gross stock – which will be reported on separately.

The results on capital services in this article show the importance of the correct treatment of the new economy assets in measures of the productive capital stock. Such results indicate the sensitivity of stock measures to the assumed life-lengths and to the deflators. Life-length assumptions and the deflator associated with an asset form the basis of weighting the stock of a particular asset in the capital services measure. Both these variables behave very differently in the new economy assets and the capital services measure is therefore sensitive to these assets.

Recently, ONS has reviewed the stock measures it produces. This is associated with two initiatives. Firstly, the completion of a new system to be used in the calculation of wealth measures of the capital stock has allowed much easier analysis and development. Capital services measures are a new product that this work has allowed. Building on this, ONS is reviewing other aspects of the model, such as the appropriate asset breakdown and the level at which modelling should take place. Secondly, the *Blue Book* in 2003 uses annual chain-linked volume indices. Volume measures are sensitive to the index used in construction, particularly to the timeliness of weights and particularly where sub-aggregates are changing rapidly, such as in ICT assets. Annually updating weights allow the chain-linked volume measures of capital services to be merged with the output measures and other input measures to calculate multi/total factor productivity.

Acknowledgement

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Table 1:
Growth in Volume Index of Capital Services, 1980–2002

Description	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002
						Annual change (%)						
By production industry												
Agriculture, forestry and fishing	–0.2	0.2	0.4	0.6	–0.4	0.1	–0.7	–2.4	–4.0	–3.2	–4.5	–2.9
Extraction – oil and gas	6.5	6.0	3.5	–2.2	–0.4	–1.2	0.2	–1.7	–4.7	–6.4	–3.1	–2.4
Other mining and quarrying	–5.5	–4.6	–4.3	–2.4	–4.9	–4.0	–4.6	–4.5	–3.7	–4.2	–3.2	–3.6
Solid and nuclear fuels; oil refining	–0.6	–2.1	0.1	10.1	2.3	–1.1	–0.1	–0.2	–0.1	–0.7	2.2	2.2
Chemicals, man–made fibres	1.2	0.4	0.1	1.2	0.5	1.5	1.4	3.6	1.2	–0.2	–0.3	–0.7
Other non-metallic minerals	–1.7	–2.1	–1.4	–1.2	1.1	2.6	0.4	1.4	2.4	–0.3	–0.3	0.1
Basic metals and metal products	–2.4	–2.9	–1.1	–1.0	–0.9	–0.1	0.4	2.0	–1.9	–3.0	–1.8	–1.4
Machinery and equipment	–0.2	–1.6	–2.3	–0.7	–0.1	–0.4	0.4	2.6	–0.8	–1.6	–0.9	–0.9
Electrical and optical equipment	–0.4	–0.5	1.6	4.0	6.4	4.7	8.9	5.3	2.0	5.4	3.6	0.1
Transport equipment	1.7	–0.5	–1.4	–0.5	1.2	1.7	6.1	3.1	2.7	0.2	2.5	0.4
Food, beverages, tobacco	1.0	0.4	0.2	–1.3	0.0	0.2	0.7	2.0	1.9	0.4	0.0	–0.2
Textile and leather products	–2.3	–1.9	–1.8	0.0	–0.8	0.6	–0.8	1.1	–1.8	–2.8	–2.9	–3.6
Pulp, paper, printing and publishing	2.8	0.4	1.7	2.7	2.8	1.0	1.5	2.9	1.7	2.6	1.9	0.7
Other manufacturing	–0.6	0.9	0.3	0.3	2.0	4.8	2.8	2.0	1.9	1.3	0.0	–0.4
Electricity	2.7	3.9	2.8	2.2	–0.5	–1.6	–0.4	–0.2	0.5	0.1	–0.2	–1.1
Gas	4.7	5.3	2.1	0.2	1.9	–3.7	–3.4	0.6	–0.4	10.2	8.1	3.5
Water	11.9	9.7	8.8	7.0	7.2	11.2	14.0	7.5	6.3	2.7	5.5	–0.6
Construction	–3.6	–3.0	0.4	4.9	1.1	–1.1	5.4	4.2	6.5	6.6	1.2	9.0
By service industry												
Motor vehicles sales and repairs	5.7	3.8	3.7	4.2	5.2	3.7	5.6	9.0	6.4	9.3	10.2	9.4
Wholesale trade	–1.8	–0.8	1.3	1.5	3.1	1.2	8.8	12.6	6.6	2.8	4.7	2.3
Retail trade	1.6	2.2	3.2	3.1	4.9	2.5	3.1	10.7	5.3	5.4	5.9	5.4
Hotels and restaurants	2.2	2.9	1.3	1.1	4.1	5.5	4.3	5.1	5.7	5.7	6.9	4.3
Rail transport	3.1	6.2	4.6	1.7	–1.2	–2.6	–3.4	–0.8	–1.0	–1.4	–0.8	–1.7
Other land transport	–6.8	–1.6	3.0	7.0	4.3	–1.6	0.9	2.2	3.7	1.5	1.9	0.7
Water transport	–2.5	–3.2	0.0	2.8	3.6	1.1	–2.5	–2.2	–0.3	8.7	–0.7	–1.0
Air transport	–2.8	8.0	22.3	33.6	–10.1	8.2	47.1	19.1	11.7	15.8	15.6	19.1
Other transport services	3.6	3.1	1.9	5.4	5.5	8.3	29.1	7.4	7.7	12.2	11.9	9.0
Post and telecommunications	0.8	–0.6	0.0	2.1	6.7	10.9	11.6	17.9	13.8	16.2	13.8	5.6
Financial intermediation	3.0	0.1	–1.3	0.8	5.1	14.3	2.3	14.1	6.5	8.6	6.4	5.9
Real estate, renting, business activities.	6.8	1.8	2.0	3.7	5.1	9.8	6.3	24.0	17.0	12.0	12.7	7.7
Public administration, etc.	1.7	1.4	2.1	1.8	1.6	0.4	0.4	0.9	1.3	0.9	1.7	2.0
Roads	3.0	4.0	4.5	4.5	3.3	2.0	0.1	1.1	0.4	0.7	0.7	1.1
Education	–3.8	0.6	1.5	1.6	1.3	1.0	2.0	2.7	1.5	1.9	3.7	3.4
Health and social work	8.5	6.3	4.1	3.8	4.1	3.1	1.1	3.1	4.5	4.2	3.0	4.4
Sewage and refuse disposal	3.7	3.2	1.7	2.0	3.9	6.1	9.8	8.7	8.5	6.7	6.0	3.4
Other services	3.6	2.4	3.1	4.7	4.4	6.3	7.2	7.3	8.3	7.7	2.5	4.7
By type of asset												
Buildings	3.3	3.4	3.1	2.7	2.3	2.4	2.4	2.4	2.3	2.7	2.3	2.2
Plant including purchased software	1.6	1.0	0.5	0.7	1.6	2.6	2.2	3.4	3.9	3.7	2.4	0.7
Computers	3.6	–4.5	–3.2	4.0	10.3	17.2	18.7	38.4	23.5	23.9	24.3	15.7
Vehicles	–3.1	–3.4	–1.2	1.6	0.2	1.5	3.1	7.5	3.0	–0.4	2.6	3.4
Intangibles excluding purchased software	1.9	1.2	1.8	3.0	2.3	1.6	1.7	4.2	1.7	2.7	0.6	3.2
Whole economy	1.8	1.5	1.9	2.3	2.3	3.4	3.6	6.2	4.7	4.8	4.4	3.2

Table 2:
Profit shares, 1990–2002

Description	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002
						Parts per thousand							
By production industry													
Agriculture, forestry and fishing	41	44	32	20	37	30	33	26	27	24	36	34	56
Extraction – oil and gas	53	66	53	40	48	46	61	47	36	39	46	45	34
Other mining and quarrying	17	12	9	8	12	10	14	11	7	8	9	7	7
Solid and nuclear fuels; oil refining	8	8	15	3	6	10	9	8	7	8	8	8	6
Chemicals, man-made fibres	32	20	17	12	32	40	33	34	35	34	33	30	27
Other non-metallic minerals	8	6	5	5	7	8	7	7	6	7	7	7	6
Basic metals and metal products	29	20	9	11	24	29	30	25	23	22	21	23	18
Machinery and equipment	18	13	11	11	17	21	17	17	17	16	14	13	12
Electrical and optical equipment	18	15	13	13	20	27	23	23	23	23	20	21	18
Transport equipment	27	20	17	60	53	31	24	26	27	27	25	24	26
Food, beverages, tobacco	29	24	17	14	28	29	25	30	29	27	29	27	26
Textile and leather products	10	11	9	7	12	13	14	12	14	13	9	8	8
Pulp, paper, printing and publishing	25	18	16	18	19	27	27	21	22	27	24	21	20
Other manufacturing	15	13	12	8	14	18	54	48	44	19	15	13	22
Electricity	48	43	37	23	59	70	63	54	59	55	55	48	41
Gas	11	11	10	11	11	10	7	11	9	7	10	10	9
Water	7	8	7	9	9	6	5	11	10	7	13	11	8
Construction	14	10	7	8	11	14	15	14	13	15	14	15	17
By service industry													
Motor vehicles sales and repairs	5	5	6	4	5	7	6	6	6	7	6	7	8
Wholesale trade	31	30	28	25	27	33	28	27	28	32	27	29	31
Retail trade	41	41	40	42	44	47	42	38	41	45	37	43	54
Hotels and restaurants	20	23	22	23	21	17	19	17	19	19	19	25	30
Rail transport	20	14	15	23	16	12	10	17	11	8	13	15	13
Other land transport	21	16	16	16	15	21	18	17	16	16	21	18	20
Water transport	9	22	5	5	7	10	6	4	4	1	5	2	4
Air transport	6	4	2	7	8	9	8	10	11	15	17	18	21
Other transport services	14	13	12	15	13	12	11	16	14	12	19	18	17
Post and telecommunications	49	60	24	38	28	67	61	64	67	79	71	88	94
Financial intermediation	60	63	50	61	52	50	52	53	51	76	80	51	61
Real estate, renting, business activities	85	103	102	105	86	92	82	77	105	114	108	138	112
Public administration, etc.	80	67	162	131	104	60	66	84	74	66	53	46	39
Roads	51	27	69	70	29	32	39	49	42	33	26	20	16
Education	26	64	69	69	45	27	28	28	29	27	30	28	35
Health and social work	12	30	30	25	27	18	19	17	18	18	20	22	22
Sewage and refuse disposal	24	18	17	21	18	11	9	19	17	14	23	18	17
Other services	36	36	34	39	35	36	36	31	36	40	37	44	45
By type of asset													
Buildings	389	515	639	653	441	206	315	375	334	257	312	315	322
Plant including purchased software	388	306	205	167	390	572	457	407	471	513	444	412	457
Computers	66	64	39	52	50	68	77	78	76	89	92	108	73
Vehicles	125	94	89	92	80	117	112	108	91	104	116	126	113
Intangibles excluding purchased software	32	21	28	36	38	37	39	32	28	36	37	39	34
Whole economy	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000

Annex

The volume index of capital services combines the long time-series of capital formation data and asset life-lengths underpinning the ONS wealth measures of capital stock with an alternative model taking the different productivity of the assets into account. Here, some background to the data and more detail regarding the VICS model is given. Data related to current work and results are available at the National Statistics website: http://www.statistics.gov.uk.

Data used to calculate VICS

The dataset consists of long back history of the volume of investment, current price investment, assumed life-lengths, and implicit price indices. The series were taken from the ONS Perpetual Inventory Method (PIM) model system, which calculates capital stock and capital consumption for the National Accounts. The PIM works at a more disaggregated level than the present series, but it is not possible to publish this microdata, as it would disclose the investments of individual businesses. The current data aggregates these series, both in current and constant prices so that the industrial disaggregation is identical to that in the ONS Supply Use Tables giving a breakdown of 36 industries.

The asset breakdown is: buildings, plant (including purchased software but excluding computers), computers, vehicles and intangibles (excluding purchased software). The five assets expand on the series currently published in the quarterly capital expenditure surveys. Series are disaggregated to the supply-use table level of 36 industries found in the annual capital formation tables. The current price datasets are calculated for 1948 onwards, consistent with published national accounts 2003. The implicit deflator, which is calculated using the current price series and the volume series, is a derived series but for some assets – for example, computers after 1995 – take the value of a published dataset.

To calculate the stock of some assets, such as buildings, several decades of investment in volume terms is necessary. Because the microdata underpinning the PIM is at a very disaggregated level, a constant price (KP) summation is used to calculate the more aggregated KP series in the spreadsheet. It is well known that constant price series are additive only in the years after the base year. Constant price summation is used in almost all areas of the national accounts, so that long time-series of constant price data stretching over several base years can be aggregated taking account of the different prices in which the series have been compiled. Table 1 in ONS (2002) gives the base years and the periods they were used.

The life-lengths assumed in the ONS wealth measures provide the average years that the assets would last for the United Kingdom. To convert these into depreciation rates, the method employed by Oulton and Srinivasan (2003, p. 77) was used for buildings, plant and machinery and vehicles. The US Bureau of Economic Analysis (Fraumeni, 1997) has done considerable work to integrate geometric decay rates into their national income and product accounts, using Hulten and Wykoff's 1981 analysis of second-hand asset prices. In computers and intangibles, the method of double-declining balance is used.

In calculating rentals, the rate-of-return is set such that the total current price capital services equals the whole economy operating surplus less that operating surplus attributable to housing, owner-occupier imputed rent and capital consumption on dwellings. HM Treasury provided the results of their work on tax-subsidy ratios – providing a time-series for each asset.

Indices of capital services

The method used in the calculation of the volume index of capital services is based largely on Oulton and Srinivasan (2003), whose paper provides an excellent analysis of the sensitivity of the index to the assumptions underlying its calculation. The first of the three steps outlined in section 2 was to aggregate the history of investments to provide a net stock. In terms of terminology, the vintage of an investment is the year of purchase of an asset.

The calculation of the net stock uses a geometric PIM.

$$(1)\quad K^i_{a,t} = I^i_{a,t} + (1-\delta^i_{a,t-1})\cdot I^i_{a,t-1} + (1-\delta^i_{a,t-2})^2\cdot I^i_{a,t-2} + \ldots$$

In equation 1, K is the volume of net stock for a particular asset a, in an industry i, t is the year under consideration, I is the investment in a year and δ is the rate of decay for the asset purchased in a particular year. It should be noted that the assumed rate of decay for an industry/asset could vary over vintages.

The rental, r, for an asset is modelled using equation 2, the Hall-Jorgenson equation (Hall and Jorgenson, 1967), where p is the price of the asset, R is a rate-of-return and TS is the tax subsidy ratio, assumed the same across industries.

$$(2)\quad r^i_{a,t} = TS_a\left[\delta^i_a\cdot p^i_{a,t} + R_t\cdot p^i_{a,t-1} - (p^i_{a,t} - p^i_{a,t-1})\right]$$

The rentals are combined with the net capital stocks to give the value-added attributable to the stock of each asset in a particular industry. The value-added shares are used as weights, w, for the VICS. In equation 3, the weights in an industry VICS is defined, though it can be generalised for any aggregate (whole economy for example, or a particular asset).

$$(3)\quad w^i_{a,t} = \frac{r^i_{a,t-1}\cdot K^i_{a,t-1}}{\sum_a r^i_{a,t-1}\cdot K^i_{a,t-1}}$$

The weights can be seen to be base period shares so that a Laspeyres VICS can be calculated, here for a particular industry, i.

$$(4)\quad VICS^i_t = \sum_a w^i_{a,t-1}\cdot\frac{K^i_{a,t}}{K^i_{a,t-1}}$$

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Capital Adjustment Patterns in Manufacturing Plants*

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A common result from altering several fundamental assumptions of the neoclassical investment model with convex adjustment costs is that investment may occur in lumpy episodes. This paper takes a step back and asks "How lumpy is investment?" We answer this question by documenting the distributions of investment and capital adjustment for a sample of over 13,700 manufacturing plants drawn from over 300 four-digit industries. We find that many plants do undergo large investment episodes; however, there is tremendous variation across plants in their capital accumulation patterns. This paper explores how the variation in capital accumulation patterns vary by observable plant and firm characteristics, and how large investment episodes at the plant level transmit into fluctuations in aggregate investment. *Journal of Economic Literature* Classification Numbers: D24, L6, E22.
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I. INTRODUCTION

Among Michael Gort's many contributions to economics is his early work using establishment-level data at the U.S. Census Bureau. Professor Gort realized early on that aggregate statistics mask important underlying

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dynamics that belie the aggregate changes, and that to truly understand the dynamics within industries, one has to examine the underlying micro data. In his 1963 paper [23], "Analysis of Stability and Change in Market Shares," Professor Gort explored the extent to which the market share of firms that make up published concentration ratios change over time. After all, if the concentration ratio for a particular industry remains high and stable over time, it does not necessarily imply that the industry is stagnant and controlled by a small handful of dominant firms. In fact, the industry could be extremely competitive with market share amongst the firms changing quite markedly, yet the published concentration ratios would not convey this information. To resolve this issue, which required access to firm-level data on market shares, Professor Gort utilized the raw micro data files at the U.S. Census Bureau. He was one of the first economists to exploit establishment-level data files at the U.S. Census Bureau for economic research. Moreover, 30 years later, Professor Gort returned to the U.S. Census Bureau to undertake a project that examined productivity growth and learning in new plants. In his 1993 paper [3], "Decomposing Learning by Doing in New Plants," coauthored with B. H. Bahk, Professor Gort examined how productivity evolves in new plants as they age. Again, this is a paper that underscores the importance of understanding the underlying microeconomic dynamics as they relate to aggregate economic changes.

Continuing in the tradition that Professor Gort helped establish, this paper also uses U.S. Census Bureau establishment-level data to gain a better understanding of an aggregate phenomenon, in this case, investment activity. This paper examines the capital adjustment patterns for a large sample of manufacturing establishments. This is an important area to examine since accurately modeling new capital investment at the micro and macro levels has proved elusive. In standard neoclassical investment models, assumptions, such as convex adjustment costs and reversibility, dictate that firms continuously and smoothly adjust their capital stock over time. While theoretically tractable, these models generally fail to adequately explain investment fluctuations [1, 8]. The disappointing empirical performance of these investment models has caused economists to reexamine the potentially unrealistic assumptions of convex adjustment costs and reversibility. Rothschild [28] argued early on that adjustment costs faced by plants and firms possess nonconvexities for a variety of reasons.[1] Another

[1] The sources of speculated nonconvexities in the cost of capital adjustment include increasing returns, the cost of the equipment, costs associated with disruption, and installation costs.

assumption in the standard models that is unrealistic is reversibility, an area that has received a great deal of attention in recent years.[2] Models which assume nonconvex adjustment costs and irreversibility possess solutions where firms occasionally adjust their capital in discrete bursts when the capital stock falls (rises) below (above) a trigger level, solutions which differ markedly from those of standard neoclassical models.[3]

While a growing number of studies suggest that capital adjustments may occur in lumpy episodes, the theoretical literature is well ahead of its empirical counterpart.[4] This is largely due to the scarcity of data sets that follow the investment process for a large number of establishments. This situation is changing as access to microeconomic data, in particular plant-level data, increases. For instance, Caballero, Engel, and Haltiwanger [10], Power [27], and Cooper, Haltiwanger, and Power [14] investigate the lumpiness of plant-level investment and its relationship to aggregate investment fluctuations using the plant-level data on investment from U.S. Census Bureau micro data files.[5] There are two main findings. First, investment by manufacturing plants is characterized by periods of intense investment activity interspersed with periods of much lower investment activity. Second, episodes of intense investment activity are responsible for a significant fraction of aggregate investment fluctuations.

This paper also examines the patterns of investment spending at the plant level and relies on the Census Bureau micro data. As compared to the Census-based research discussed above, this paper is more descriptive. The goal of this paper is to present a series of stylized facts that will serve as benchmarks for investment models. In particular, the goal of this paper is not simply to show whether investment is lumpy or not, but instead to focus on how the distributions of investment and capital adjustment vary by plant characteristics (e.g., industry, size, age, and ownership) and by level of micro-unit aggregation (plant, line-of-business, and firm). Finally, the paper relates the evidence on micro-level lumpiness to aggregate investment fluctuations.

[2] Reviews of investment models with irreversibility include Pindyck [26], Dixit [17], and Dixit and Pindyck [18].

[3] Other underlying assumptions in neoclassical models are that capital is homogeneous and capital depreciates geometrically. Feldstein and Rothschild [22] discuss the unrealistic nature of homogeneous capital and geometric decay, and how changing these assumptions can result in lumpy investment patterns.

[4] The literature which examines labor adjustments is more mature. The importance of large proportional adjustments in employment at the establishment level has been documented by Hamermesh [24], Davis and Haltiwanger [16], and Caballero, Engel, and Haltiwanger [10].

[5] In addition to the Census Bureau microeconomic data studies, there are a number of other studies that examine machine replacement at the micro level. Rust [29] examines replacement investment with bus engines, and Cooper and Haltiwanger [13] model retooling in automobile assembly plants.

We first examine the patterns of capital accumulation within plants and focus on the magnitude of capital adjustments at annual frequencies. We find:

(1) Many plants occasionally alter their capital stocks in lumpy fashions. Over half of the plants in our sample experience a 1-year capital adjustment of at least 37%. While many manufacturing plants experience episodes of intense investment activity, 80% of the plants in a given year change their net capital stock by less than 10%. These relatively small changes account for 52% of total sample investment.

Whether or not capital adjustment is "lumpy" depends on to what it is compared. To help quantify what "lumpy" means, we compare the results from the sample of plants to those generated by simulated investment models, where the simulations include possible S, s behavior by including trigger and target levels. The larger are the estimated trigger levels in the simulations, the "lumpier" is capital adjustment. We find:

(2) The simulation models that best fit the observed capital adjustment patterns are those that possess trigger levels substantially above and below zero. That is, the simulation results that best fit the observed data are those in which plants mainly invest when the difference between the desired and actual capital stocks is substantially different. Otherwise, plants usually invest in small amounts, amounts that could be related to replacement and maintenance investment.

Although many plants do experience a large investment episode, perhaps our most striking finding is the tremendous variance across plants in their capital adjustment patterns. We find:

(3) With respect to plant characteristics, smaller plants, plants that undergo a change in organizational structure (e.g., ownership change), and plants that switch industries have lumpier investment patterns.

Although investment is conducted at the establishment level, investment decisions are made at the firm level. Hence, while investment may be relatively volatile at the establishment level, investment may be smoothed at the firm level, which may be consistent with the large literature on the role of firm finance constraints. In fact, we find:

(4) Plant-level capital accumulation patterns are considerably lumpier than those computed at the line-of-business level, and the line-of-business level capital accumulation patterns are noticeably more discrete than those at the firm level.

Whether or not investment is lumpy also influences models of aggregate investment. Increasing attention has recently been placed on unraveling aggregate fluctuations by examining the distribution of micro changes (e.g., [7, 9, 10, 14, 16]). Bertola and Caballero [5] model firms making investment decisions in an uncertain environment and when investment is irreversible. In this model, firms do not continually invest, but invest in lumps; hence, aggregate fluctuations in investment are partially attributable to changing proportions of the population undergoing large investment episodes. To shed light on this issue, we examine how plant-level changes in capital and investment transmit to aggregate fluctuations in investment, focusing particularly on the role of investment spikes. We find:

(5) Large investment projects in a small number of plants greatly impact aggregate investment. For our sample, 25% of expenditures on new equipment and structures goes into plants that are increasing their real capital stock by more than 30%. However, these plants make up only 8% of the sample. For the population as a whole, investment is highly skewed. In 1977 and 1987 the 500 largest investment projects accounted for 35.7 and 32.1% of total manufacturing investment.

(6) Periods of large aggregate investment are due, in part, to changes in the frequency of plants undergoing large investment episodes, though not necessarily large percentage changes in capital adjustments.

The paper proceeds as follows. Section II describes the data and the patterns of capital adjustment observed in our data sets, and provides results of simulations used to benchmark the empirical patterns. This section also examines how capital adjustment patterns vary by producer characteristics and by level of aggregation. Section III discusses the correlation between large capital adjustments and fluctuations in aggregate investment. Section IV provides summary analysis.

II. PLANT-LEVEL CAPITAL ACCUMULATION PATTERNS

In this section we examine the patterns of plant-level investment and capital growth, focusing especially on those periods when plants undergo large changes in their capital stocks. The section presents some basic statistics on capital growth rates and investment, and examines how these patterns vary by plant characteristics such as industry, plant size, and unit of aggregation (e.g., plant, line of business, firm). Before proceeding with a description of the basic patterns, we briefly describe the data. A more thorough discussion of the data set can be found in Doms and Dunne [20].

The information on annual investment and capital growth is constructed from a panel data set of manufacturing plants for the period 1972–1988. The establishment-level data are drawn from the Longitudinal Research

Database (LRD), which is maintained at the U.S. Census Bureau and contains establishment-level production data from the Annual Survey of Manufacturers (ASM). The main data set contains a balanced panel of establishments from the LRD and covers the period 1972–1988. The balanced nature of the panel ensures that capital stocks for plants can be constructed using the perpetual inventory method. The resulting data set includes 13,702 manufacturing establishments. This sample is small relative to the manufacturing population, which ranges from 312,000 to 360,000 plants over the sample period. However, while the sample coverage in terms of number of establishments is relatively small, these establishment are on average quite large and account for a significant fraction of manufacturing investment, production, and employment. Table I presents some basic characteristics of this data sample. The establishments in the sample averaged over 500 employees and accounted for 55.4–61.1% of manufacturing investment, employed 39.3–44% of manufacturing workers, and produced 47.4–53.8% of manufacturing output over the 1972–1988 period. While not reported in this paper, we have also constructed a larger data set that allows for establishment births and deaths. In general, the results reported below hold qualitatively for plants that do not span the entire time period. These results are reported in Doms and Dunne [20].

In order to measure plant-level capital growth rates, a capital series must be developed for each plant. In this paper we use the perpetual

TABLE I
Sample Coverage by Year

Year	Investment coverage (%)	Labor coverage (%)	Production coverage (%)	Average employment
1973	58.6	43.5	53.0	598.7
1974	60.1	44.0	53.8	600.4
1975	58.8	44.0	52.7	551.8
1976	56.9	44.2	54.0	570.3
1977	57.1	43.5	53.1	588.2
1978	55.4	43.3	53.2	608.1
1979	59.3	43.2	53.7	622.2
1980	60.5	42.7	52.7	601.9
1981	60.5	43.2	52.6	595.8
1982	57.7	42.2	50.3	548.7
1983	61.1	41.9	51.1	534.7
1984	57.3	42.8	51.8	558.7
1985	60.8	42.9	50.6	547.6
1986	58.2	42.5	50.1	530.5
1987	56.7	40.2	48.3	520.7
1988	58.1	39.3	47.4	514.3

inventory method. The capital stock in period t for plant i, $K_{i,t}$, is defined as

$$K_{i,t} = K_{i,t-1}(1 - \delta) + I_{i,t}, \tag{1}$$

where δ represents the depreciation rate and $I_{i,t}$ is current period investment. The rate of depreciation, δ, is estimated for each three-digit industry by imbedding the depreciation parameter within a production function. The parameters of the production function are estimated simultaneously with the parameters of the investment stream (see Doms [19] for details). Utilizing the above measure for the capital stock we construct net capital growth rates analogous to the employment growth rates of Davis and Haltiwanger [16]. The growth rate of capital for plant i at time t is computed as

$$GK_{i,t} = \frac{I_{i,t} - \delta K_{i,t-1}}{0.5 \cdot (K_{i,t-1} + K_{i,t})}. \tag{2}$$

For each plant in our sample, we compute $GK_{i,t}$ for every year from 1973 to 1988.[6,7]

Figure 1 presents two distributions, the density of $GK_{i,t}$ and the density of $GK_{i,t}$ weighted by $I_{i,t}$. The figure shows that 51.9% of plants in a year increase their capital stock by less than 2.5%, while 11% of plants in a year increase their capital stock by more than 20%. However, the few plants that do undergo large changes contribute significantly to the level of aggregate investment. The weighted distribution shows that 25% of investment is in plants increasing their capital stock by more than 25%. At the other end of the distribution, 19.2% of investment is occurring in plants changing their capital stock by less than 2.5%.

Figure 1 shows that the distribution of investment is skewed, with a small number of plants accounting for a relatively large share of investment. While this is present in our subsample of data, it is also true in the establishment population as a whole. Table II gives the share of total investment in 1977 and 1987 accounted for by the top 100 investing plants, top 500 investing plants, top 1000 investing plants, etc. Also given in Table II are the analogous figures for ranked employment and output. This table

[6] The Annual Survey of Manufacturers stopped collecting the book value of capital data in 1989, as well as other investment related data, making it difficult to compute capital stocks after 1988.

[7] Unfortunately, the above expression ignores early retirements in the construction of the capital stock. The LRD does contain some data on retirements, but these data appear to contain significant errors. The constructed growth rate is therefore a relative measure of new capital accumulation net of depreciation.

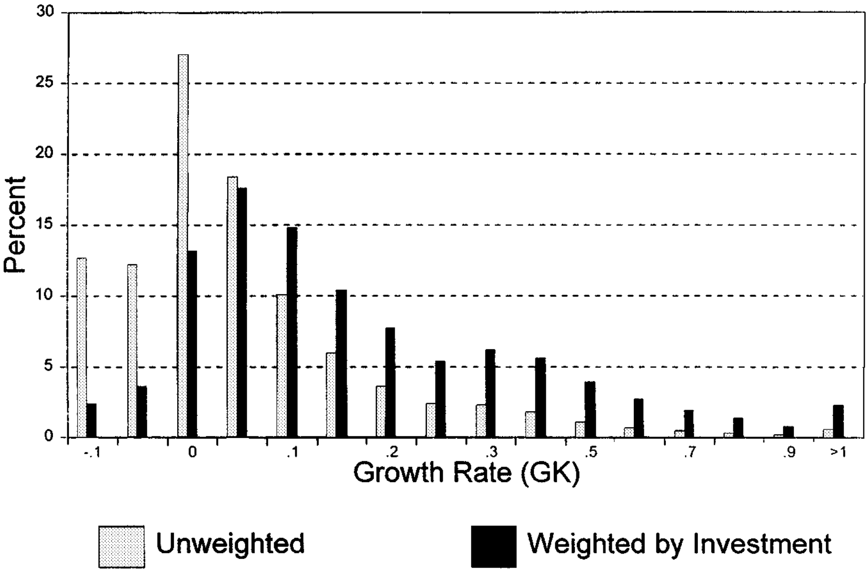

FIG. 1. Capital growth rate (GK) distributions: Unweighted and weighted by investment.

is based on the *entire* manufacturing establishment population. The overall message is relatively clear. A small number of plants account for a large fraction of investment. In 1977 and 1987, 18.2 and 16.2% of total manufacturing investment was accounted for by the top 100 plants, respectively. In contrast, the top 100 plants accounted for a substantially smaller fraction of ouput (9.0%) and employment (5.9%). Note that 100 plants make up only 0.028% of the entire population. The bottom line is that in a cross section a small number of investment "projects" account for a substantial fraction of aggregate investment. While this cross-sectional result is suggestive of "lumpy" investment, it does not provide any information on the within-plant investment patterns over time. It is a description of these within-plant patterns of investment and capital adjustment that we turn to next.

To examine the within-plant capital accumulation patterns, we construct two sets of ranks to describe the distributions of capital growth and investment at the plant level. The first measure constructs a ranked distribution of capital growth rates for a plant. For each plant in the balanced panel, we rank their capital growth rates from highest to lowest, so that their maximum growth rate is rank 1 and their lowest growth rate is rank 16. Throughout this paper, the rank 1 growth rate is denoted by MAXGK. Figure 2a presents the means and medians of these ranked

TABLE II
Share of Investment, Employment, Shipments, and Capital Accounted for by the Top Plants in Each Category

	1987 Census of manufactures: 358,567 plants			
	Investment	Employment	Output	Capital stock
Top 100 plants	.16204	.06344	.10077	.11888
Top 500 plants	.32154	.14057	.23031	.28882
Top 1000 plants	.41268	.18982	.30819	.38497
Top 5000 plants	.64769	.36233	.52581	.60963
Top 10000 plants	.74987	.47020	.62994	.70622
Top 25000 plants	.86863	.64043	.77045	.83002
Top 50000 plants	.93531	.77445	.86831	.90761
	1977 Census of manufactures: 350,648 plants			
	Investment	Employment	Output	Capital stock
Top 100 plants	.18172	.05932	.09005	.12883
Top 500 plants	.35657	.14584	.21638	.29359
Top 1000 plants	.44948	.20269	.29398	.39090
Top 5000 plants	.67240	.38958	.51551	.62407
Top 10000 plants	.76821	.50301	.62389	.72395
Top 25000 plants	.87931	.67753	.77172	.84548
Top 50000 plants	.94131	.80945	.87187	.91819

growth rates, so the first set of bars in Fig. 2a shows the mean and median MAXGK. The next set of bars shows the means and medians of the second largest growth rates, and so on. These bars indicate that the mean MAXGK slightly exceeds 46%, while the median is 36%. The means and medians drop off significantly after rank 1. Figure 2a illustrates that many plants experience a few periods of intense capital growth and many periods of relatively small capital adjustment: of the 16 capital growth rate ranks, 12 possess means or medians between -10 and $+10\%$.

Besides the growth rates of the capital stock, we are also concerned with episodes of investment that account for a large share of a plant's total investments. Figure 2b plots the mean proportion of total 16-year investment that occurs in each year. For instance, the leftmost bar represents that the average plant experiences a 1-year investment episode that accounts for 24.5% of its total real investment spending over the 16-year interval. The secondary growth rate accounts for 14.7%, and the third highest accounts for 10.9% of investment. This implies that, on average, half of a plant's total investment over the 1973–1988 period was performed in just three years. An important point is that while a significant portion of investment occurs in a relatively small number of episodes, plants still invest in every period.

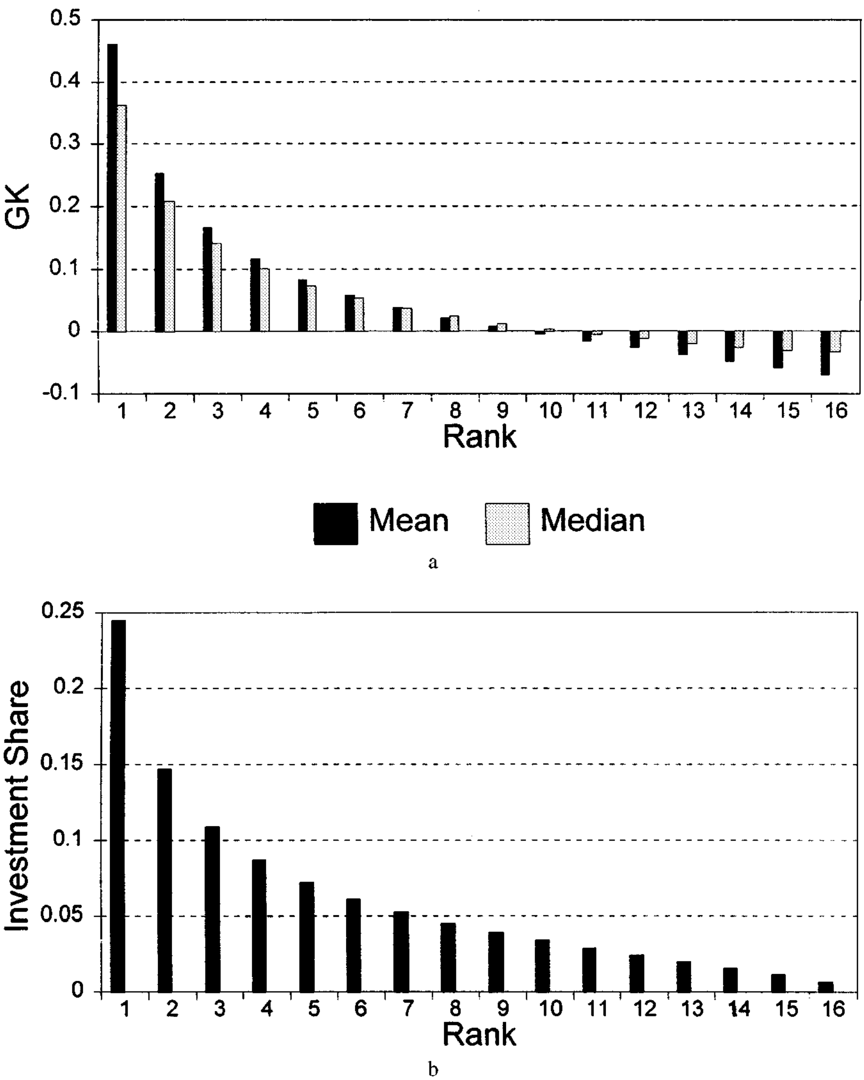

FIG. 2. Capital growth rates (*GK*) by rank, means, and medians. (b) Mean investment shares by capital growth rate rank.

What does Fig. 2a and b say about whether investment is "lumpy" or not? By construction, these figures slope down, and it is difficult to tell if, for instance, the data generating the figures come from something as simple as a Gaussian white noise process or whether the data are truly representative of a "lumpy" process. To benchmark our results, we compare our empirical results to simulations of simple capital investment models that include the possibility of lumpy adjustment episodes. Although the simulations do not formally test particular investment models, the simulations do provide a convenient benchmark to view our results. The following model was kindly provided by Jeffrey Campbell.

Let $k^*_{i,t}$ denote the optimum level of the logarithm of the capital stock of plant i at time t if the plant faced no frictions in adjustments, frictions that might arise from nonconvex adjustment costs or irreversibilities. Let $k_{i,t}$ be the actual capital stock. In the simulations that follow, we assume that the optimum level of capital, $k^*_{i,t}$, follows a random walk of the form

$$k^*_{i,t} = k^*_{i,t-1} + \varepsilon_{i,t}, \qquad \varepsilon_{i,t} \sim N(\mu, \sigma^2). \tag{3}$$

The disturbance $\varepsilon_{i,t}$ is i.i.d. across time and plants. Let $z_{i,t} = k_{i,t} - k^*_{i,t}$ be the difference between the frictionless optimum and the actual capital stock. The investment decision for a plant follows that of a general S, s model, where the trigger levels are denoted by U and L and target levels by u and l, such that $L \leq l \leq u \leq U$. For instance, if $U = u = l = L = 0$, then there are no frictions and plants will always invest to their optimal frictionless level of capital and capital adjustment would be normally distributed. However, if the target levels do differ from zero, then there will be periods when no investment takes place, that is, periods in which $z_{i,t-1} + \varepsilon_{i,t}$ lies within the U, L band. If $z_{i,t-1} + \varepsilon_{i,t} < L$, then the plant will invest up to l. Likewise, if the optimum level of capital falls sufficiently, $z_{i,t-1} + \varepsilon_{i,t} > U$, then the plant will disinvest to u. We modify this basic friction model by adding replacement investment since some investment always takes place in our sample of establishments. Replacement investment, $r_{i,t}$, is uniformly distributed and is independent across time and plants.

The simulations are performed with 1000 plants and are run for 300 periods. The last 16 observations for each plant are taken and the capital adjustments are ranked, just as they are ranked with the real data. The parameters of the simulations are calibrated to minimize the mean squared error between the simulated values and the real values of the ranked capital adjustments. For nearly all of the simulations presented in this paper, the values of the replacement and innovation parameters are nearly identical; for the innovation parameters, $\mu = 0.05$ and $\sigma = 0.18$. The mean value of replacement investment is 0.05 with a standard deviation of 0.005–0.02.

Figure 3 reproduces Fig. 2a with the results of two simulations. The first simulation is the best fitting simulation with frictionless adjustment, $U = u = l = L = 0$. What is perhaps most striking about the frictionless adjustment simulation is its symmetry, which stands in stark contrast to the asymmetry in the real data. Additionally, the frictionless simulation does not drop as quickly or have as many periods with low capital accumulation activity as in the real data. The second simulation presented in Fig. 3 introduces frictions, that is, the target and trigger levels are allowed to deviate from zero. The friction parameters that produce the best results are $L = -0.34$, $l = -0.05$, $u = 0.20$, and $U = 0.22$. What is most striking about this simulation is how the simulated values sharply fall after the first rank and then stay much closer to 0, as in the real data. In fact, the mean squared error between the simulations and the actual data falls from 0.129 for the frictionless model to 0.012 for the friction model.

The results in Fig. 3 are for the entire sample of establishments that span the sample period. What is also striking is how the results in Fig. 2a vary by other observable plant characteristics, such as size. Figure 4 presents mean ranks by size quartile, where plants are ranked by their mean employment over the sample period. The basic result is that smaller plants have higher maximum growth rates than the largest plants. We again perform the simulations for these four plant size categories, and the parameter that changes the most is the trigger level L, which goes from -0.37 for the smallest quartile to -0.20 for the largest quartile, a significant difference. One of many possible reasons why smaller establish-

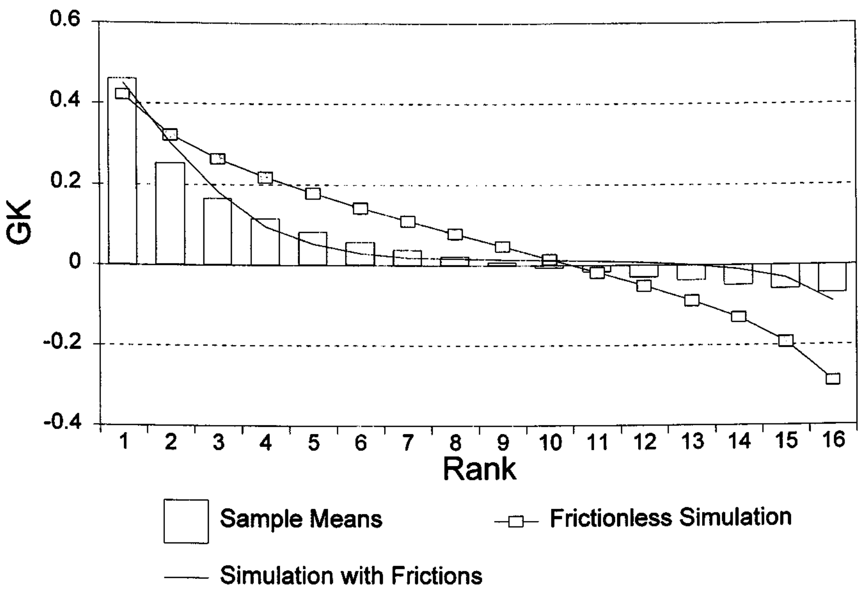

FIG. 3. Mean capital growth rates (GK) by rank, sample means, and simulated values.

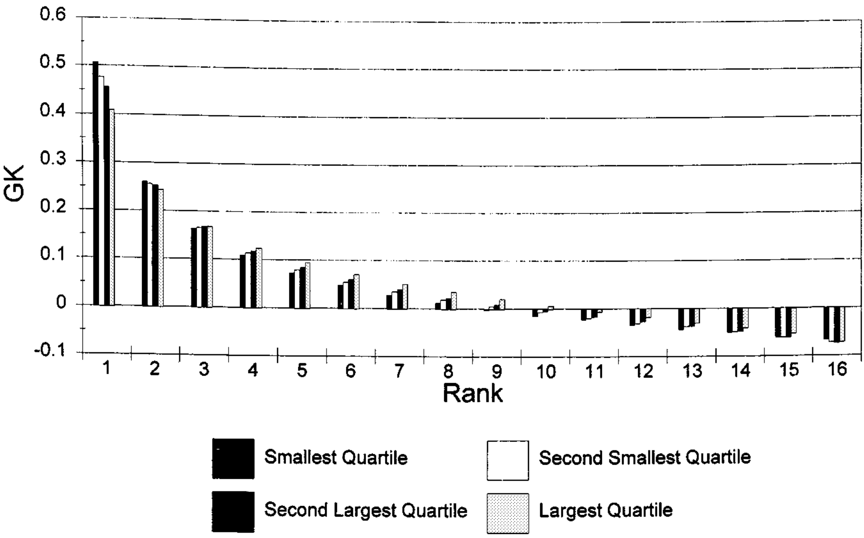

FIG. 4. Mean capital growth rates (*GK*) by rank and by size quartile.

ments may have higher trigger levels than larger establishments may be the indivisible nature of capital equipment; buying a single new machine at a smaller plant may represent a large share of its capital stock, so that its investment pattern may appear "lumpy." Large plants employing many machines may have smoother investment patterns because a single machine is a very small share of its capital stock. Additionally, one could view a large plant as a collection of smaller operations producing a range of products. These multiproduct operations may face less variable sales due to the fact they produce a number of different products, and hence their investment may be smoother as well.

Up to this point the unit of observation has been the plant; however, there are many arguments which suggest that the investment decisions of a plant are made at the divisional or firm level. Additionally, there are reasons why firms may smooth investment across plants. To examine how capital adjustment patterns vary by plant and firm, we construct capital adjustment ranks at the plant, the two-digit industry line-of-business level, and the firm level. The sample used to construct the plant, line-of-business, and firm statistics is a subset of the balanced panel. First, only those plants that remain with a single firm for at least 14 out of the 16 years are used. Second, only those plants that belong to firms with at least three plants are kept. Given these requirements, only 5822 plants out of the 13,702 plants in the balanced panel remain, representing 648 firms and 955

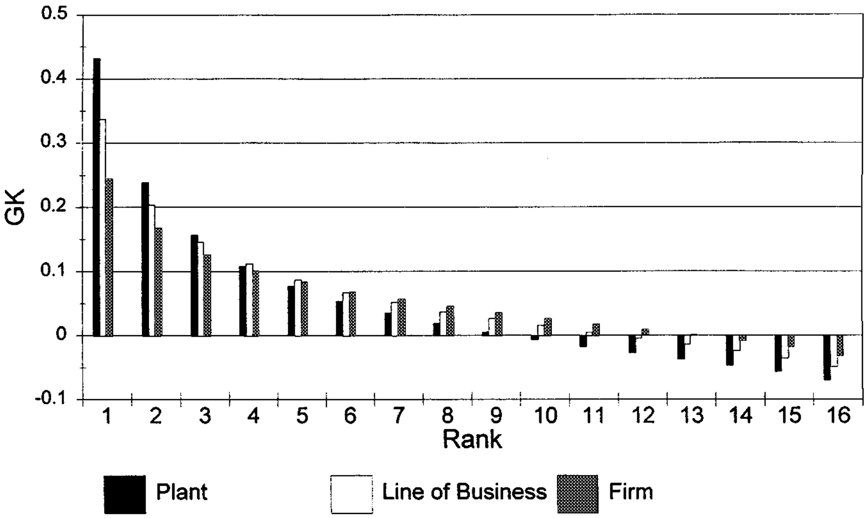

FIG. 5. Mean capital growth rates (GK) by plant, line of business, and firm.

lines of business. Note, however, that these plants make up 72.5% of the balanced panel investment.

The results of this exercise are presented in Fig. 5.[8] Basically, the higher is the level of aggregation, the smoother is the capital adjustment rank distribution. Examining the height of the largest capital adjustment episode, the mean MAXGK for plants is 0.432, and it falls to 0.336 for the line of business, and falls even further to 0.245 for firms. Again, simulations for these three distributions are run, and the estimate for the trigger level L changes from -0.35 for plants, to -0.19 for the line of business, and to -0.10 for the firm. This finding of smoother investment at the firm level may be consistent with the results reported by Cummins, Hassett, and Hubbard [15]. Note, however, that the asymmetry of the capital growth rate distribution still persists even at the firm level.

The analysis, so far, shows considerable across plant variation in capital growth rates, suggesting some plants experience relatively smooth changes in their capital stocks while other plants undergo sizable jumps in their capital stocks. A possible explanation of the differences in size is that for

[8] The basic results in Fig. 5 also hold for the investments distribution. Examining the height of the largest investment spike episode, the mean plant maximum investment share is 24%. This is quite close to that reported in Fig. 2b for the entire balanced sample. The mean maximum plant investment share drops to 17.1% at the line-of-business level and to 15.8% at the firm level. The bottom line is that firm-level investment patterns appear to be considerably smoother than plant-level investment patterns.

some industries investment is inherently lumpy because of the nature of the capital goods (which could arise due to the indivisibility of large machines), while for other industries it may be easier to adjust capital more smoothly.[9] To examine this possibility, we model MAXGK for a plant as a function of size, controlling for industry and other effects.

We estimate a regression model using all plants in our balanced panel. Our plant-level measure of capital lumpiness is the maximum single year capital growth rate (MAXGK), which our simulations show to be closely related to the magnitude of the trigger levels. Also, we have constructed other variables that characterize a plant's capital adjustment patterns, and arrive at the same qualitative results. The regressions include controls for both plant and firm size. Plant size is modeled using a set of dummy variables representing plant-size quintiles. The quintiles go from smallest to largest, with the quintile representing the largest plants omitted. The firm size variables are similarly defined. Two variables are included to capture potential changes in organizational structure and production mix that may affect capital accumulation patterns. The first variable is a dummy variable indicating whether a plant has changed ownership during the sample period. The second variable is a dummy variable which indicates whether the plant changes the two-digit industry in which the plant operates. Two age variables are included to capture differences in the age of plants that entered the panel in 1972. Finally, the regressions are all run with four-digit industry dummy variables. To conserve on space, the industry coefficients are not reported in the tables.

The second column of Table III reports the regression results. The starkest result is the strong inverse relationship between plant size and MAXGK. Smaller plants have considerably larger spikes, even after controlling for industry and other plant characteristics. Alternatively, there is no discernible pattern in the firm size coefficients. The two variables which capture change in ownership and change in industry indicate that plants which undergo ownership changes or switch industries experience somewhat larger MAXGKs. This is consistent with the view that organizational and industry changes lead to changes in plant-level operations which affect capital accumulation decisions. In terms of the simulation models, changes in ownership structure and industry may be indicators of discrete changes

[9] Doms and Dunne [20] report considerably more industry-level detail. For example, in the case of investment spikes, we find that 10% of industries (four-digit SIC) have maximum investment spikes under 0.20, 80% have maximum investment spikes between 0.20 and 0.30, and the remaining 10% have maximum investment spikes exceeding 0.30. Hence, the investment spike patterns observed in Fig. 2b are also present in a wide range of four-digit SIC industries. The same finding would be true for the capital growth rate distributions. Figure 2a is qualitatively similar to the growth rate distributions for a large number of industries.

TABLE III
Capital Growth Rate Regression: MAXGK Is the Dependent Variable

440 4-digit Industry Controls	Included
16 Year Dummies in which MAXGK occurs	Included
Plant Size Quintiles (Smallest to Largest)	
1st Quintile	.319 (.012)
2nd Quintile	.178 (.011)
3rd Quintile	.109 (.010)
4th Quintile	.056 (.010)
5th Quintile	omitted
Firm Size Quintiles (Smallest to Largest)	
1st Quintile	.007 (.010)
2nd Quintile	.037 (.009)
3rd Quintile	.013 (.009)
4th Quintile	.014 (.014)
5th Quintile	omitted
Industry Change Indicator	.031 (.011)
Ownership Structure Change Indicator	.040 (.006)
1963 Age Dummy	−.086 (.009)
1967 Age Dummy	−.049 (.011)
Mean of Dependent Variable	.461
Number of Observations	13072
R^2	.211

in the desired capital level. On the other hand, older plants have generally smaller than average capital growth rate spikes. This last result is consistent with the Jovanovic's [25] model of industry evolution that predicts that the variance of growth should decline as firms age.

The regression coefficients provide basic evidence of how capital growth varies with observable plant characteristics. However, on the whole, the plant and industry characteristics explain relatively little of variation in the standard deviation of capital growth or in the size of MAXGK. The amount of variation explained by plant and industry controls is about 20%. In general this lines up with the results reported by Davis and Haltiwanger [16], who report R^2s of similar magnitudes for employment growth regressions.

III. AGGREGATE INVESTMENT FLUCTUATIONS

This paper has so far focused on the predominance of large capital adjustments in plants and the variation across plants in their capital adjustment patterns. Increasing attention has recently been placed on unraveling aggregate fluctuations by examining the distribution of micro changes (e.g., [7, 9, 10, 12, 14, 16]). In this section, we present some basic summary statistics on the relationship between aggregate fluctuations in investment, the uniformity of changes in capital, and the frequency of large capital adjustments.

Using the balanced panel, which annually accounts for approximately 58% of aggregate investment, we compute the frequency of plants that have their MAXGK and MAXI (the maximum investment share) in a given year. Figure 6 presents these frequencies in addition to aggregate real investment over the period 1973–1988. There are several items to note. The first is that the correlation between MAXI and aggregate investment is 0.59, which is significant at the 99% level. The correlation between MAXGK and aggregate investment, however, is not statistically significant. This is due primarily to the high frequency of MAXGKs in 1973 and 1974 which is not reflected in the aggregate data.

Figure 6 conveys that aggregate fluctuations are correlated with the frequency of plants undergoing large investment episodes. An alternative way to summarize the relationship between aggregate investment and lumpy episodes is to see if investment is more skewed or concentrated in high investment periods. To address this issue, we compute a Herfindahl index for investment in each year and plot this series in Fig. 7 along with

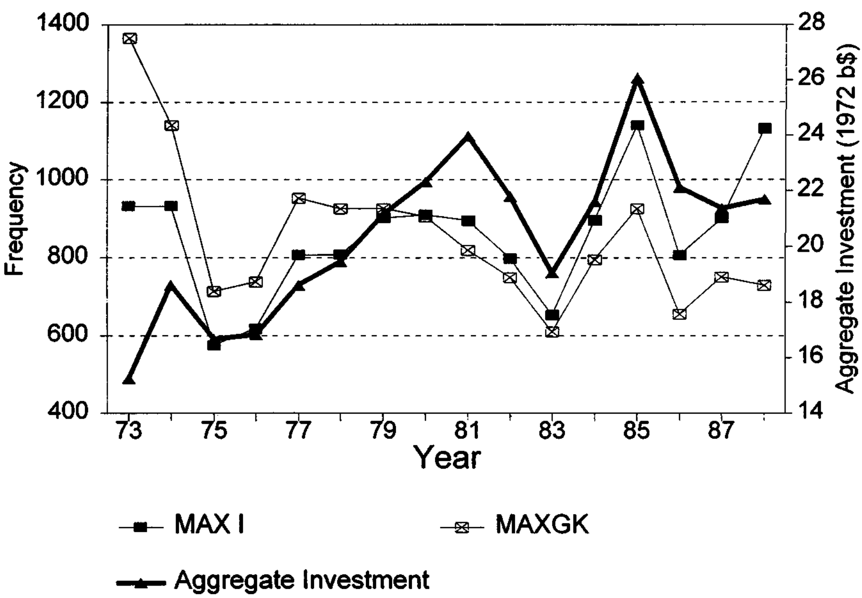

FIG. 6. Aggregate investment and frequency of plant spikes.

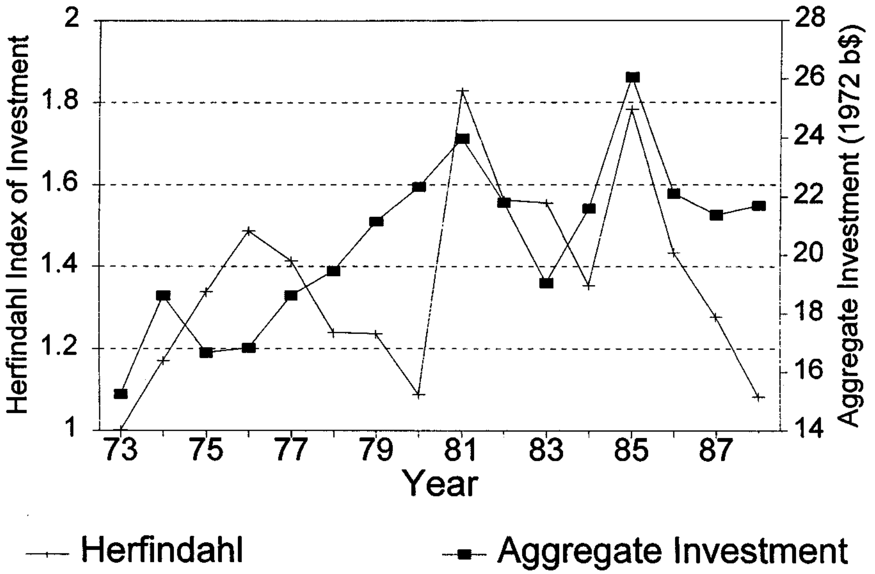

FIG. 7. Aggregate investment and the Herfindahl index of investment.

the aggregate investment series for the period 1973–1988.[10] In general, the series move together. The correlation between the two series is 0.450 and is significant at the 90% level. An interesting feature to note in Fig. 7 is that in 1980 and 1988 there are periods of relatively high aggregate investment in which there are relatively low Herfindahls. However, the two highest Herfindahls are in the 2 years with the highest aggregate investment.

IV. CONCLUSION

The objective of this paper is to present a series of stylized facts concerning the capital accumulation patterns for a large set of manufacturing plants. Although this paper is primarily descriptive in its examination of plant-level investment behavior, the facts presented here are quite striking and raise a number of issues. We have shown that many manufacturing plants do indeed alter their capital stocks in lumpy fashions, and these large adjustments do account for a significant portion of a plant's

[10] The Herfindahl index for investment is constructed as $\sum (I_i/TI)^2$, where I_i is investment in plant i and TI is aggregate investment. The Herfindahl is just the sum across all plants of the squared investment shares.

total capital expenditures and aggregate investment. However, we also find tremendous heterogeneity in the capital accumulation patterns across plants, finding that the degree of lumpiness of capital adjustment varies considerably across plants. These facts certainly raise the question of whether traditional representative agent models based on convex costs of adjustment are adequate enough to examine the dynamics of investment and capital accumulation.

That said, there are many features of the capital accumulation process that have not been addressed in the paper. One key aspect we have not examined is the within-plant timing pattern of investment. In particular, we have said little about what happens to a plant before a spike and, more importantly, what happens to a plant after a spike. To shed some light on this issue, Fig. 8 presents the mean growth rates of capital over a 5-year period surrounding the maximum capital growth spike, MAXGK. One can see that both before and after a spike, plants return to a much lower level of investment spending. This confirms the view that large capital growth episodes are interspersed with periods of relatively modest capital growth at the plant level. The specifics of investment timing are addressed more fully in papers by Caballero, Engel, and Haltiwanger [10], Cooper, Haltiwanger, and Power [14], and Power [27]. Importantly, Cooper, Haltiwanger, and Power [14] show that the probability of an establishment undergoing an investment spike increases in the time since the last investment spike. This line of research lends support to the notion that plants wait until their

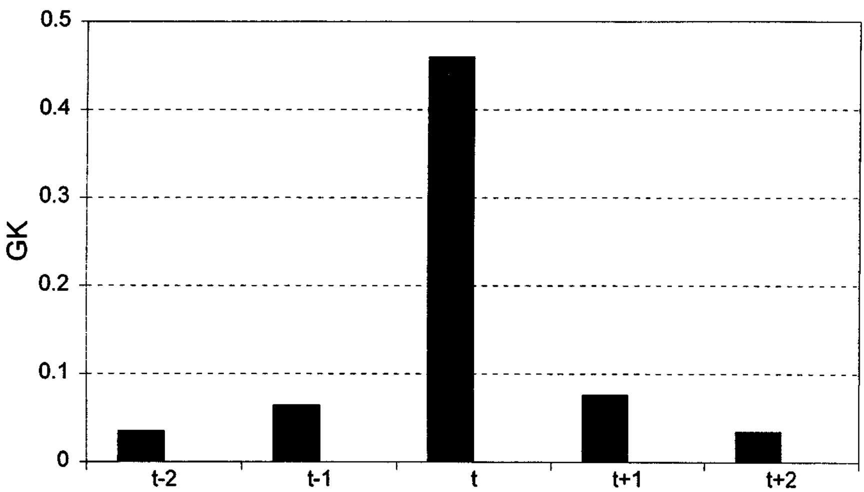

FIG. 8. Mean pre- and post-spike capital growth rates (GK).

actual capital stock deviates from the desired stock by a threshold before they invest.

In closing, this paper has described the patterns of capital accumulation using micro data on manufacturing establishments from the U.S. Census Bureau. This type of work builds on the tradition which Michael Gort helped establish almost three and half decades ago in his research using firm-level Census data from the 1940s and 1950s, and it highlights the importance of access to and development of micro data resources in understanding the underlying micro dynamics of aggregate data fluctuations.

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